Mathematical Conversations

Gilbert Strang: The Changing Face of Applied Mathematics >>>

An interview of Gilbert Strang by Y.K. Leong

Gilbert Strang is a prominent scholar in applied mathematics and an active promoter of mathematics and mathematical education in the United States. He has numerous contributions to numerical analysis, wavelets and signal processing. He is also well-known for his textbooks on linear algebra and applied mathematics at the undergraduate and advanced levels. He is editor of many well-known journals and has given invited lectures throughout the world. He has received numerous honors and awards and is a Fellow of the American Academy of Arts and Sciences. He has served on many committees, in particular, as President of the Society of Industrial and Applied Mathematics (SIAM) in 1999 and 2000. He is currently Chair of the US National Committee on Mathematics for 2003-2004. He has been Professor of Mathematics at MIT since 1970.

He has been closely associated with NUS, having served as a member of the University's International Advisory Panel from 1998 to 2001 during a period of reorganization. The Editor of *Imprints* interviewed him on 18 July 2003 at the Institute for Mathematical Sciences when he visited NUS as a guest of the Institute and the Singapore MIT Alliance from 15 to 19 July 2003. The following is an edited transcript of the interview in which he spoke about e-learning and teaching, applied mathematics in the service of society and the changing landscape of applied mathematics.

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A landscape so changed Courtesy Gilbert Strang

I: MIT recently launched an ambitious long term project to make its courses freely available on the web. How has this project influenced the teaching of mathematics at MIT?

S: Generally, the MIT OpenCourseWare is an e-learning project. I don't think anybody knows exactly how the course materials on the web are going to be used. It was by chance that I had videos taken of my linear algebra lectures (I teach a lot of linear algebra courses). About two years ago, they were video taping the freshman physics course when I came into the same room for the next hour. So, I said, "Maybe you can just keep the cameraman in the room for another hour and we will have a video tape of the linear algebra lecture". They got some financial support for it and they did it. As a result, my linear algebra course is one of the few that have videos on the web. It's absurd that I am now a movie star on OpenCourseWare, which is available at http://ocw.mit.edu. That course was in the first group and now 500 courses are available. I have two others, an applied mathematics course and a wavelets course. The linear algebra course was an early one.

From early feedback on the number of hits around the world, Hong Kong was leading in the number of users. It may be different now. I think students like to see videos which are more alive than course notes. Your question asks about e-learning and mathematics. E-learning has to be alive! The lectures have to somehow involve the student. The real challenge is for students to be a part of the learning process and not just passive viewers.

I: Is there any significant influence on people's teaching?

S: It makes teaching interesting in a new way. After my turn, they took videos of really good lectures at the Mathematics Department. For example, Professor Arthur Mattuck had given lectures for years on calculus and differential equations. I was happy that he was video taped. He is always very well organized. We are also trying to see if there is a way for us to have students answer simple questions in the class, either by pressing a button or in some other ways. You want them to follow the lecture closely, instead of sitting there till the end and then leaving. The key is active learning.

I: You were President of the Society for Industrial and Applied Mathematics (SIAM) some time ago. Could you share with us some of your experiences, challenges and achievements during your presidency?

S: I was President for two years in 1999 and 2000. For a year before, I was the President-Elect, and Past President the year after, so it was a four-year commitment. Quite a lot of time, but I enjoyed it. Applied Mathematics in the U.S. has been upgrading its efforts in connecting with Washington. I think Singapore is amazing because you are always well connected to the needs of society and to the goals of the ministry. In the U.S., we were far from Washington in the past (well, still mostly so). Mathematicians are thinking more about their own work than about the goals of the Science Minister. But now, there is more and more input to the National Science Foundation, to the Office of Management and Budget, to the Senate, to the House. This is a crucial step towards addressing the question of what applied mathematics can contribute to society. It is a challenge, and a happy experience, to speak to the House, to the Senate and to their staffs about mathematics.

One of my special experiences was getting two new activity groups started: one in the mathematics of Life Sciences, and one in Imaging Science. In SIAM, the activity groups have concentrated conferences every two years. It is really important for applied and pure mathematics to recognize what are the new directions and to help the new areas. Some problems grow more and some less, and the rate of changes varies.

The action moves into new areas like the life sciences. It's really important for SIAM to be a part of those areas. It does a lot of conference organising, but the job of the President and the Council of the society is to see what new actions and what new work the society should be doing. That depends on where applied mathematics is moving.

The Society needs to establish a better image of mathematics. The engineers are always there to sell their ideas, and the physicists and biologists are also there. You have to demonstrate the ability of mathematics to contribute and you have to show that mathematicians are needed. Of

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course, that has to be proved by what actually happens so that the engineers or the biologists can see the contribution from mathematics.

I: The term "applied mathematics" seems to change over time and place. What does it mean nowadays?

S: There are different parts of applied mathematics. One person is probably not naturally prepared to work in all these parts. An important part is problem modeling. Given the physical problem, produce differential equations that capture the essential facts of the physical problem. Modeling is a big step and I am always impressed by people who do that well. Modeling may not be rigorous but people who do it will agree when it is done well. There is definitely a standard of good and bad in modeling, but it is not a theorem-proof standard. There is an intuitive agreement that, yes, the important parts of the problem are correctly accounted for.

I: How much rigor do we need to maintain in modeling?

S: Maybe not rigor in a sense of proving, but it has to be convincing. The expert in the field might say, "Ah-ha, you missed this important aspect of the problem" and that will affect the solution. There is a standard but it's not exclusively logical. After the model is created comes computing. "Computational science" is now the word that includes both these aspects. You understand the area of application and you can carry through the computing to obtain and understand the answer. But often that is shared. One person understands the application area and the other person says, "Just give me the equation and I will work on the solution."

I: Does that mean the applied mathematician must have two modes of thought?

S: Yes, if he or she is complete. The modeling mode and, we could say, the computing mode.

I: Not many people are able to do that.

S: Unfortunately so but it doesn't have to be. It's good if it is all in one person because then that person sees the full picture. But very often, say in linear algebra, there are many people who will ask, "I have a very large symmetric matrix. How do I find the five smallest eigenvalues?" To the expert in numerical algebra this is a question independent of where the matrix comes from. There is a further stage of creating good software that could be permanent and that other people could use. So there you really need to understand computer science. So, we have the modeling stage, the solution stage and the software stage. The third stage seems far from classical applied mathematics, but it is part of the whole process.

I: So it seems that the computer is really indispensable?

S: It is now, yes. In the past people could get amazing information about the solution by analytical methods and by perturbation methods. That was done by the mandarins of applied mathematics. Now it's more democratic and it's a larger world. Engineers are now part of the modeling world, using algorithms and sometimes creating algorithms. For example, the finite element method is a famous example of an idea that was already in the mathematics papers by Courant and Feng Kang but was really brought to importance by the engineers.

I had an interesting conversation with Nick Trefethen in Oxford on whether mathematicians have made the key contribution in scientific computing. He convinced me that it was probably so. He went down the list of main algorithms that are really dominant. One example would be the Fast Fourier Transform that is used almost everywhere. It came from, well, maybe statisticians (in John Tukey's case) and from mathematicians (Gauss was the first). There is a whole range of crucial algorithms in the twentieth century like the fast multipole methods, stability problems for finite differences, and applied mathematicians have been at the heart of the progress of those methods.

I: Who provided the key ideas?

S: I have never before made the list that Trefethen has: what are the key ideas and who brought them to birth? I like engineering activities and certainly engineers deserve credit. Wavelets is a good example where work in applied mathematics has changed the direction of some area of pure mathematics. Wavelets grew out of signal processing, which is a big user of mathematics. Wavelets have their own unique direction and depth in the pure mathematics sense. Progress in that area continues. That is also well reflected in the Institute's programs and in the wavelet center here.

I: It seems that applied mathematicians would need to work with somebody else.

S: Often, yes. The new problems and excitement are now in the life sciences and biology. You need the biologists, of course. They know what the problems are but they need the mathematicians too. The border between pure and applied mathematics becomes quite permeable. It is very satisfying to contribute to the solutions that other people want. That really is a good feeling. And pure mathematicians often see those patterns and relationships that are the fundamental to mathematics.

I: Is it necessary to give special training in order to make somebody into a good applied mathematician?

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S: Well, I don't know. I think that the modeling ability is partly genetic and in-born. The computing skill comes partly from experience – actually doing the computing, getting the answers and improving the algorithms. It is a field where the supply is far behind the demand. The demand is everywhere and I would like to see more and more students having the fun of contributing to a team of scientists.

I: Do you think that the ability to model is unique and that not everybody can do it?

S: Unique, yes. But it must be possible to teach something to those of us who have not got it. I suppose I may be feeling about modeling the way so many outsiders feel about mathematics: that it is a mystery. How could people do this? But mathematicians don't see it as a mystery. They see it as a natural ability. Probably modeling should be just another ability that could be developed. But the very best modelers seem to be natural and not taught.

I: Do you consider modeling to be more of an art?

S: Yes it is. The computing part (the algorithmic part as well) takes on its own creativity. It is an area where you need to experiment and if you have an algorithm, you prove that it is good by using it. A paper that just suggests an algorithm that is never tried is not really acceptable. The proof is in the solution.

I: What are the big problems in applied mathematics?

S: One of the very big areas has been computational fluid dynamics, CFD, and in the extreme case, turbulence, where the parameters are approaching the departure from a smooth flow. That is a big problem and there is good progress. There will always be wonderful problems. But I may not be the right person to name all the big problems.

I: Is turbulence a computational problem?

S: Well, you could say that it is partly computational. You need to invent new methods. There is another big area currently: multi-scale computation. Typically you create a mesh. You have a step-size close to the time scale or the length scale of the problem. But if the problem has a length scale that extends from the size of an atom or molecule to the size of the body or the size of the earth, you cannot use the molecular length scale. How do you make the computation at the molecular length scale acceptable for a model of the heart? Somehow you have to find a smart way to see the effects of very, very small scale events at the macro scale.

I: What about artificial intelligence? It used to be a hot topic.

S: Yes, it did. Maybe it did count as applied mathematics. Applied mathematics certainly goes outside of the mathematics department. But maybe artificial intelligence has remained even further outside. There is such a big range of hard problems. I'm not really qualified to talk about artificial intelligence.

I: There seems to be a gap between pure mathematicians and applied mathematicians. Some people think that it is due to the differences in philosophy and culture between them. How do we bridge this gap?

S: First of all, there has to be mutual respect. Even pure mathematics itself (or applied mathematics) is too big for anybody to bridge, much less to bridge all of mathematics. I think if we have mutual respect and cooperation and an open mind when looking for places to contribute, then the gap is not important. It is the solving of problems that is important. I feel that SIAM (which is an applied mathematics society) and the American Mathematical Society (which is more pure) have included everything, and they are cooperating more and more. I would expect that there is similar cooperation at the Institute here.

I: Should applied mathematicians have the same faith that physicists often have in their usually intuitive, ad hoc and non-rigorous methods in solving many of their mathematical problems?

S: You see physicists getting very clever, usually correct, results. There are definite differences between physicists and applied mathematicians. And probably many physicists are not as close to the computer as many applied mathematicians would be.

I: We shall round up with this question: Should mathematical rigor be imposed at the undergraduate level?

S: For teaching, I think maybe not. I am not sure if I want you to print that! The idea with examples is where students learn. My goal in teaching is always ideas and examples and not proofs. Of course, the mathematics has to be correct, but to prove that it is correct at every step, I don't see as important in teaching. Those students who naturally have that ability will go that way, but students who have other abilities should be allowed to go in the directions natural for them. That's my thought about teaching. If we want mathematics to be chosen by good students, to be liked by students and to be helpful to society, we need a broad view of our subject.

I: Thank you so much for sharing your ideas with us.

S: It is my pleasure to be in Singapore and to see the Institute developing in a great way