Interview with Abel Prize Recipient Lennart Carleson

Martin Raussen and Christian Skau

Lennart Carleson is the recipient of the 2006 Abel Prize of the Norwegian Acdemy of Science and Letters. On May 22, 2006, prior to the Abel Prize celebration in Oslo, Carleson was interviewed by Martin Raussen of Aalborg University and Christian Skau of the Norwegian University of Science and Technology. The interview was later shown on Norwegian television. The first two questions in the interview, and their answers, were originally phrased in three Scandinavian languages: Norwegian, Danish, and Swedish. They are here translated into English. This interview originally appeared in the September 2006 issue of the *European Mathematical Society Newsletter*.

R & S: *On behalf of the Norwegian and Danish mathematical societies, we want to congratulate you on winning the Abel Prize for 2006.*

This year we commemorate the 100th centenary of the death of the Norwegian dramatist and poet Henrik Ibsen. He passed away on the 23rd of May just a stone's throw away from this place. The longest poem he ever wrote is called "Balloon letter to a Swedish lady" and it contains a verse which reads as follows:

"—aldri svulmer der en løftning
av et regnestykkes drøftning
—ti mot skjønnhet hungrer tiden—"

Translated into English this becomes:

"—never arises elation
from the analysis of an equation
—for our age craves beauty—"

Without drawing too far-reaching conclusions, Ibsen seems to express a feeling shared by many people, i.e., that mathematics and beauty or art are opposed to each other, that they belong to different spheres. What are your comments to this view?

Carleson: I do not think that Ibsen was very well-oriented about beauty in mathematics, which you certainly can find and enjoy. And I would even maintain that the beauty of many mathematical arguments can be easier to comprehend than many modern paintings. But a lot of mathematics is devoid of beauty. Maybe particularly in modern mathematics, where problem areas have often gotten extremely complex and complicated, with the result that the solution can only be formulated on several hundreds of pages. And that can scarcely be called beautiful. But in classical mathematics you find many striking theorems and arguments that hit you as something really original. It is reasonable to use the term beauty for those.

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From left to right: Lennart Carleson, Martin Raussen, and Christian Skau.

R & S: *Mathematicians all over Scandinavia are proud of counting one of their own among the very first recipients of the Abel Prize. How would you characterize and evaluate Scandinavian, and particularly Swedish, mathematics in an international perspective?*

Carleson: I think that Scandinavia does quite well in this respect. In Sweden, we have a fine new generation of young mathematicians. And I think it looks very much alike in the other Scandinavian

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Lennart Carleson receives the Abel Prize for 2006 from Queen Sonja.

countries. It is difficult to perceive a new Abel on the horizon, but that is probably too much to hope for.

R & S: *Could you please characterize the unique contribution that the Finnish/Swedish school of Lindelöf, M. Riesz, Carleman, R. Nevanlinna, Phragmen, Beurling, and Ahlfors brought to analysis in the first half of the 20th century, which was formative and decisive for your own contribution to hard analysis?*

Carleson: In your list, you miss another Scandinavian mathematician: J. L. Jensen. The importance of "Jensen's inequality" can hardly be exaggerated. He and Lindelöf started the Scandinavian school, building of course on Riemann's approach to complex analysis rather than that of Cauchy-Weierstrass; Nevanlinna and Carleman continued, followed by Ahlfors and Beurling, a remarkable concentration of talent in Scandinavia. My lecture tomorrow will give more details.

Mathematical Achievements in Context

R & S: *Abel first thought that he had solved the general quintic by radicals. Then he found a mistake and subsequently he proved that it was impossible to solve the quintic algebraically. The famous and notoriously difficult problem about the pointwise convergence almost everywhere of L^2-functions, which Lusin formulated in 1913 and actually goes back to Fourier in 1807, was solved by you in the mid-1960s. We understand that the prehistory of that result was converse to that of Abel's, in the sense that you first tried to disprove it. Could you comment on that story?*

Carleson: Yes, of course. I met the problem already as a student when I bought Zygmund's book on trigonometric series. Then I had the opportunity to meet Zygmund. He was at Harvard in 1950 or 1951. I was at that time working on Blaschke products, and I said maybe one could use those to produce a counterexample. Zygmund was very positive and said "of course, you should do that." I tried for some years and then I forgot about it before it again came back to me. Then, in the beginning of the 1960s, I suddenly realized that I knew exactly why there had to be a counterexample and how one should construct one. Somehow, the trigonometric system is the type of system where it is easiest to provide counterexamples. Then I could prove that my approach was impossible. I found out that this idea would never work; I mean that it couldn't work. If there were a counterexample for the trigonometric system, it would be an exception to the rule.

Then I decided that maybe no one had really tried to prove the converse. From then on it only took two years or so. But it is an interesting example of "to prove something hard, it is extremely important to be convinced of what is right and what is wrong". You could never do it by alternating between the one and the other because the conviction somehow has to be there.

R & S: *Could we move to another problem, the so-called Corona problem that you solved in 1962? In this connection, you introduced the so-called Carleson measure, which was used extensively by other mathematicians afterwards. Could you try to explain why the notion of the Carleson measure is such a fruitful and useful notion?*

Carleson: Well, I guess because it occurs in problems related to the general theory of BMO and H^1-spaces. I wish this class of measures had been given a more neutral name. In my original proof of the Corona problem, the measures were arc lengths on the special curves needed there. Beurling suggested that I should formulate the inequality for general measures. The proof was the same and quite awkward. Stein soon gave a natural and simple proof and only then the class deserved a special name.

R & S: *I'll move to another one of your achievements. Hardy once said that mathematics is a young man's game. But you seem to be a counterexample; after you passed sixty years of age, you and Michael Benedicks managed to prove that the so-called Hénon map has strange attractors exhibiting chaotic behaviour. The proof is extremely complicated. It's a tour de force that took many years to do. With this as a background, what is your comment on mathematical creativity and age?*

Carleson: I guess and hope that you don't get more stupid when you get older. But I think your stamina is less, your perseverance weakens (keeping lots of facts in your mind at the same time).

Probably this has to do with the circulation of the blood or something like that. So I find it now much harder to concentrate for a long period. And if you really want to solve complicated problems, you have to keep many facts available at the same time.

Mathematical Problems

R & S: *You seem to have focused exclusively on the most difficult and profound problems of mathematical analysis. As soon as you have solved any one of these, you leave the further exploration and elaboration to others, while you move on to other difficult and seemingly intractable problems. Is this a fair assessment of your mathematical career and of your mathematical driving urge?*

Carleson: Yes, I think so. Problem solving is my game, rather than to develop theories. Certainly the development of mathematical theories and systems is very important but it is of a very different character. I enjoy starting on something new, where the background is not so complicated. If you take the Hénon case, any schoolboy can understand the problem. The tools also are not really sophisticated in any way; we do not use a lot of theory.

The Fourier series problem of course used more machinery that you had to know. But that was somehow my background. In the circles of dynamical systems people, I always consider myself an amateur. I am not educated as an expert on dynamical systems.

R & S: *Have there been mathematical problems in analysis that you have worked on seriously, but at which you have not been able to succeed? Or are there any particular problems in analysis that you especially would have liked to solve?*

Carleson: Yes, definitely. There is one in dynamical systems, which is called the standard map. This is like the Hénon map but in the area-preserving case. I spent several years working on it, collaborating with Spencer for example, but we never got anywhere.

If you want to survive as a mathematician, you have to know when to give up also. And I am sure that there have been many other cases also. But I haven't spent any time on the Riemann hypothesis… and it wouldn't have worked either.

Characterization of Great Mathematicians

R & S: *What are the most important features, besides having a good intellectual capacity of course, that characterize a great mathematician?*

Carleson: I don't think they are the same for everybody. They are not well defined really. If you want to solve problems, as in my case, the most important property is to be very, very stubborn. And also to select problems which are within reach. That needs some kind of intuition, I believe, which is a little closer to what we talked about initially,

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Lennart Carleson

about beauty. You must somehow have a feeling for mathematics: What is right, what is wrong, and what is feasible. But, of course, there are many other mathematicians who create theories and they combine results into new buildings and keep other people working. It is a different kind of a mathematician. I don't think you should try to find a simple formula for people.

R & S: *For several decades, you have worked hard on problems that were known to be exceptionally difficult. What drove you and what kept you going for years, with no success guaranteed? What drives a person to devote so much energy to an arcane subject that may only be appreciated by a handful of other mathematicians?*

Carleson: Yes, that's a big issue. Stubbornness is important; you don't want to give up. But as I said before, you have to know when to give up also. If you want to succeed you have to be very persistent. And I think it's a drive not to be beaten by stupid problems.

R & S: *Your main research contribution has been within mathematical analysis. What about your interest in algebra and topology/geometry?*

Carleson: Geometry is of course very much part of the analysis. But I have no feeling for algebra or topology, I would say. I have never tried to… I should have learned more!

Mathematics of the Future

R & S: *What do you consider to be the most challenging and exciting area of mathematics that will be explored in the twenty-first century? Do you have any thoughts on the future development of mathematics?*

Photo: Scanpix.

Abel laureates Peter Lax (2005), Lennart Carleson (2006), and Jean-Pierre Serre (2003).

Carleson: Yes, of course I have had thoughts. Most of the influence comes from the outside. I think we are still lacking a good understanding of which kind of methods we should use in relation to computers and computer science. And also in relation to problems depending on a medium-sized number of variables. We have the machinery for a small number of variables, and we have probability for a large number of variables. But we don't even know which questions to ask, much less which methods to use, when we have ten variables or twenty variables.

R & S: *This leads to the next question. What is the significance of computers in mathematics? Is it mainly checking experimentally certain conjectures? Or is it completing proofs by checking an enormous amount of special cases? What are your thoughts on computers in mathematics?*

Carleson: There are a few instances that I have been involved with. I had a student, Warwick Tucker, who proved that the Lorenz attractor exists. The proof was based on explicit computations of orbits. And in that case you could get away with a finite number of orbits. This is very different from the Hénon map, where you could never succeed in that way. You could never decide whether a parameter was good or bad. But for the Lorenz attractor he actually proved it for the specific values that Lorenz had prescribed. Because it is uniformly expanding, there is room for small changes in the parameter. So this is an example of an actual proof by computer.

Of course then you could insist on interval arithmetics. That's the fine part of the game so to say, in order to make it rigorous for the people who have very formal requirements.

R & S: *But what about computers used, for instance, for the four-color problem, checking all these cases?*

Carleson: Probably unavoidable, but that's okay. I wouldn't like to do it myself. But it's the same with group structures, the classification of simple groups, I guess. We have to accept that.

R & S: *The solution of the 350-year-old Fermat conjecture, by Andrew Wiles in 1994, uses deep results from algebraic number theory. Do you think that this will be a trend in the future, that proofs of results which are simple to state will require a strong dose of theory and machinery?*

Carleson: I don't know.

The striking part in the proof of the Fermat theorem is the connection between the number theory problem and the modular functions. And once you have been able to prove that, you have moved the problem away from what looked like an impossible question about integers, into an area where there exists machinery.

Career. Teachers.

R & S: *Your CV shows that you started your university education already at the age of seventeen and that you took your Ph.D. at Uppsala University when you were twenty-two years old. Were you sort of a wunderkind?*

Carleson: No, I didn't feel like a wunderkind.

R & S: *Can you elaborate about what aroused your mathematical interests? And when did you become aware that you had an exceptional mathematical talent?*

Carleson: During high school I inherited some books on calculus from my sister. I read those but otherwise I didn't really study mathematics in any systematic way. When I went to university it was natural for me to start with mathematics. Then it just kept going somehow. But I was not born a mathematician.

R & S: *You already told us about your Ph.D. advisor, Arne Beurling, an exceptional Swedish mathematician, who is probably not as well known as he deserves. Could you characterize him as a person and as a researcher in a few sentences? Did he have a lasting influence on your own work?*

Carleson: Yes, definitely. He was the one who set me on track. We worked on the same type of problems but we had a different attitude towards mathematics. He was one of the few people about whom I would use the word genius. Mathematics was part of his personality somehow. He looked at mathematics as a piece of art. Ibsen would have profited from meeting him. He also considered his papers as pieces of art. They were not used for education and they were not used to guide future researches. But they were used as you would use a painting. He liked to hide how he found his ideas. If you would ask him how he found his result, he would say a wizard doesn't explain his tricks.

So that was a rather unusual education. But of course I learned a lot from him. As you said, he has never been really recognized in a way which he deserves.

R & S: *Apart from Arne Beurling, which other mathematicians have played an important part in your development as a mathematician?*

Carleson: I have learnt from many others, in particular from the people I collaborated with and in particular from Peter Jones. I feel a special debt to Michel Herman. His thesis, where he proved the global Arnold conjecture on diffeomorphisms of the circle, gave me a new aspect on analysis and was my introduction to dynamical systems.

R & S: *You have concentrated your research efforts mainly on topics in hard analysis, with some spices from geometry and combinatorics. Is there a specific background for this choice of area?*

Carleson: I don't think so. There is a combinatorial part in all of the three problems we have discussed here. And all of them are based on stopping time arguments. You make some construction and then you stop the construction, and you start all over again.

R & S: *This is what is called renormalization?*

Carleson: Yes, renormalization. That was something I didn't learn. Probability was not a part of the Uppsala school. And similarly for coverings, which is also part of the combinatorics.

R & S: *Which mathematical area and what kind of mathematical problems are you currently the most interested in?*

Carleson: Well, I like to think about complexity. I would like to prove that it's harder to multiply than to add.

R & S: *That seems to be notoriously difficult, I understand.*

Carleson: Well, I am not so sure. It's too hard for me so far.

R & S: *You have a reputation as a particularly skillful advisor and mentor for young mathematicians; twenty-six mathematicians were granted a Ph.D. under your supervision. Do you have particular secrets on how to encourage, to advise, and to educate young promising mathematicians?*

Carleson: The crucial point, I think, is to suggest an interesting topic for the thesis. This is quite hard since you have to be reasonably sure that the topic fits the student and that it leads to results. And you should do this without actually solving the problem! A good strategy is to have several layers of the problem. But then many students have their own ideas. I remember one student who wanted to work on orthogonal polynomials. I suggested that he could start by reading Szegö's book. "Oh, no!" he said, "I don't want to have any preconceived ideas."

Publishing Mathematics

R & S: *I would like to move to the organization of research. Let's start with the journal* Acta Mathematica. *It is a world famous journal founded by Gösta Mittag-Leffler back in 1882 in Stockholm as a one-man enterprise at that time. It rose very quickly to be one of the most important mathematical journals. You were its editor in chief for a long period of time. Is there a particular recipe for maintaining* Acta *as a top mathematical journal? Is very arduous refereeing most important?*

Carleson: It is the initial period that is crucial, when you build up a reputation so that people find it attractive to have a paper published there. Then you have to be very serious in your refereeing and in your decisions. You have to reject a lot of papers. You have to accept being unpopular.

R & S: *Scientific publication at large is about to undergo big changes. The number of scientific journals is exploding and many papers and research results are sometimes available on the Internet many years before they are published in print. How will the organization of scientific publication develop in the future? Will printed journals survive? Will peer review survive as today for the next decades?*

Carleson: I've been predicting the death of the system of mathematical journals within ten years for at least 25 years. And it dies slowly, but it will only die in the form we know it today. If I can have a wish for the future, I would wish that we had, say, 100 journals or so in mathematics, which would be very selective in what they publish and which wouldn't accept anything that isn't really finalized, somehow. In the current situation, people tend to publish half-baked results in order to get better promotions or to get a raise in their salary.

The printing press was invented by Gutenberg 500 years ago in order to let information spread from one person to many others. But we have completely different systems today which are much more efficient than going through the printing process, and we haven't really used that enough.

I think that refereeing is exaggerated. Let people publish wrong results, and let other people criticize. As long as it's available on the 'net it won't be any great problem. Moreover, referees aren't very reliable; it doesn't really work anyway. I am predicting a great change, but it's extremely slow in coming. And in the meantime the printers make lots of money.

Research Institutions

R & S: *I've just returned from a nice stay at the Institute Mittag-Leffler, which is situated in Djursholm, north of Stockholm; one of the leading research institutes of our times. This institute was, when you stepped in as its director in 1968, something that I would characterize as a sleeping beauty. But you turned it into something very much different, very active within a few years. By now around thirty mathematicians work together there at any given time but there is almost no permanent staff. What was the inspiration for the concept of the Institute Mittag-Leffler as it looks today? And how was it* *possible to get the necessary funds for this institute? Finally, how would you judge the present activities of the institute?*

Carleson: To answer the last question first, I have to be satisfied with the way it worked out and the way it continues also. I just hope that it can stay on the same course.

In the 1960s, there was a period when the Swedish government (and maybe also other governments) was willing to invest in science. There was a discussion about people moving to the United States. Hörmander had already moved and the question was whether I was going to move as well. In this situation, you could make a bargain with them. So we got some money, which was of course the important part. But there was a rather amusing connection with the *Acta*, which is not so well known. From Mittag-Leffler's days, there was almost no money in the funds of the academy for the Mittag-Leffler Institute. But we were able to accumulate rather large sums of money by selling old volumes of the *Acta*. Mittag-Leffler had printed large stocks of the old *Acta* journals which he never sold at the time. They were stored in the basement of the institute. During the 1950s and early 1960s one could sell the complete set of volumes. I don't remember what a set could be sold for, maybe US$1,000 dollars or so. He had printed several hundred extra copies, and there were several hundred new universities. If you multiply these figures together you get a large amount of money. And that is still the foundation of the economy of the institute.

R & S: *A bit later, you became the president of the International Mathematical Union, an organization that promotes international cooperation within mathematics. This happened during the cold war and I know that you were specifically concerned with integrating Chinese mathematics at the time. Could you share some of your memories from your presidency?*

Carleson: Well, I considered my main concern to be the relation to the Soviet Union. The Chinese question had only started. I went to China and talked to people in Taiwan, and to people in mainland China. But it didn't work out until the next presidential period, and it simply ripened. The main issue was always whether there was to be a comma in a certain place, or not, in the statutes.

It was somehow much more serious with the Russians. You know, they threatened to withdraw from international cooperation altogether. The IMU committee and I considered that the relation between the West and the East was the most important issue of the International Mathematical Union. So that was exciting. Negotiations with Pontryagin and Vinogradov were kind of special.

R & S: *Did these two express some anti-semitic views also?*

Wreath ceremony at the Abel monument in Oslo.

Carleson: No, not officially. Well they did, of course, in private conversation. I remember Vinogradov being very upset about a certain Fields Medal being given to somebody, probably Jewish, and he didn't like that. He said this is going to ruin the Fields Medal forever. Then I asked him if he knew who received the first Nobel Prize in literature. Do you? It was a French poet called Sully Prudhomme; and that was during a period when Tolstoy, Ibsen, and Strindberg were available to get the prize. Well, the Nobel Prize survived.

Mathematics for Our Times

R & S: *You wrote a book,* Matematik för vår tid *or* Mathematics for Our Times, *which was published in Sweden in 1968. In that book, you took part in the debate on so-called New Mathematics, but you also described concrete mathematical problems and their solutions. Among other things you talked about the separation between pure and applied mathematics. You described it as being harmful for mathematics and harmful for contact with other scientists. How do you see recent developments in this direction? What are the chances of cross-fertilization between mathematics on the one side and, say, physics, biology, or computer science on the other side? Isn't computer science somehow presently drifting away from mathematics?*

Carleson: Yes, but I think we should blame ourselves; mathematics hasn't really produced what we should, i.e., enough new tools. I think this is, as we talked about before, really one of the challenges. We still have lots of input from physics, statistical physics, string theory, and I don't know what. I stand by my statement from the 1960s.

But that book was written mostly as a way to encourage the teachers to stay with established values. That was during the Bourbaki and New Math period, and mathematics was really going to pieces, I think. The teachers were very worried, and they had very little backing. And that was somehow the main reason for the book.

R & S: *If you compare the 1960s with today, mathematics at a relatively elevated level is taught to many more people and other parts of the subject are emphasized. For example the use of computers is now at a much higher state than at that time, where it almost didn't exist. What are your main points of view concerning the curriculum of mathematics at, say, high school level and the early years of university? Are we at the right terms? Are we teaching in the right way?*

Carleson: No, I don't think so. Again, something predictable happens very slowly. How do you incorporate the fact that you can do many computations with these hand-computers into mathematics teaching?

But in the meantime, one has also expelled many things from the classroom which are related to the very basis of mathematics, for example proofs and definitions and logical thinking in general. I think it is dangerous to throw out all computational aspects; one needs to be able to do calculations in order to have any feeling for mathematics.

You have to find a new balance somehow. I don't think anybody has seriously gotten there. They talk a lot about didactics, but I've never understood that there is any progress here.

There is a very strong feeling in school, certainly, that mathematics is a God-given subject. That it is once and for all fixed. And of course that gets boring.

Public Awareness

R & S: *Let us move to public awareness of mathematics: It seems very hard to explain your own mathematics to the man on the street; we experience that right now. In general pure mathematicians have a hard time when they try to justify their business. Today there is an emphasis on immediate relevance, and it's quite hard to explain what mathematicians do to the public, to people in politics, and even to our colleagues from other sciences. Do you have any particular hints on how mathematicians should convey what they are doing in a better way?*

Carleson: Well, we should at least work on it; it's important. But it is also very difficult. A comment which may sound kind of stupid is that physicists have been able to sell their terms much more effectively. I mean, who knows what an electron is? And who knows what a quark is? But they have been able to sell these words. The first thing we should try to do is to sell the words so that people get used to the idea of a derivative, or an integral, or whatever.

R & S: *As something mysterious and interesting, right?*

Carleson: Yes, it should be something mysterious and interesting. And that could be one step in that direction, because once you start to talk about something you have a feeling about what it is. But we haven't been able to really sell these terms. Which I think is too bad.

R & S: *Thank you very much for this interview on behalf of the Norwegian, the Danish, and the European Mathematical Societies!*