Leonid Bunimovich: Stable Islands, Chaotic Seas >>>

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Interview of Leonid Bunimovich by Y.K. Leong

Leonid Bunimovich has contributed to the fundamental understanding of dynamical systems and made important applications of probability and statistics to geophysical hydrodynamics, neuroscience, operations research, statistical mechanics, mathematical biology and numerous other scientific areas.

Bred in the great Russian tradition of probability, statistics and mathematical physics of the well-known and influential mathematician Yakov Sinai, Bunimovich began his research interests in Moscow University and quickly developed his own original and independent approaches to various problems in genetics, geophysics, biology, statistical physics and other scientific areas outside mathematics – problems that he encountered in his scientific journey that crossed the high seas and great continents. Even before the political convulsions that shook and broke up the former Soviet Union in 1991 took place, his seemingly chaotic path in academia finally found a niche for him in the School of Mathematics of the Georgia Institute of Technology. Holding the Regents' Professorship, he was the director of the Southeast Applied Analysis Center at Georgia Institute of Technology, and continues as the director of its successor program – the Applied & Biological Contemporary Mathematics Program. He is a winner of the Humboldt Prize and serves on the editorial boards of numerous leading international journals on applied mathematics and mathematical physics. He is actively engaged in organizational work for many scientific meetings around the world. He has traveled widely to major research centers as visiting professor and been invited to speak at major scientific meetings around the world.

Bunimovich was in the organizing committee of the Institute's program on *Dynamical chaos and non-equilibrium statistical mechanics: From rigorous results to applications in nano-systems* held from 1 August to 30 September 2006. He was interviewed on 17 August 2006 by Y.K. Leong on behalf of *Imprints*. The following is an edited and enhanced version of the transcript of the interview in which he traced the unusual scientific odyssey that took him physically to many places and scientifically into numerous disciplines which speak the language of mathematics. He gives us a first-hand account of scientific discovery and a bird's eye view of the enigmatic landscape at the interface of physics and mathematics which underlies the tantalizing field of chaotic dynamics. He also gives us an insight into the politics of a monolithic ideology that impeded the development of the biological sciences in the vastest political empire that dominated the world in the 20th century.

Imprints: Your PhD in Moscow was in probability and mathematical statistics while your Doctor of Science was in theoretical and mathematical physics. Was there a switch of research interest or was it more of a "natural" transition of research interest? What motivated it?

Leonid Bunimovich: It was not a switch of research interest. I graduated from the department of probability theory and was a student of Professor Sinai who is the major person in dynamical systems and one of the greatest mathematicians of our time. He was a student of Kolmogorov. I was working on the statistical properties of dynamical systems. This again goes back to Kolmogorov. The general view is that there are random phenomena, and there are deterministic phenomena, but in his short paper of 1958, Kolmogorov built a bridge between the world of random systems and the world of deterministic systems. This was the major event that started the revival of ergodic theory, which is the statistical theory of dynamical systems. At that time, it was considered to be essentially a finished area of mathematics. All of a sudden it started to evolve and grow enormously. This is considered to be one of the major developments in science, not only in mathematics, in the 20th century. My thesis was in stochasticity of dynamical systems, where a new mechanism of stochasticity, later called a mechanism of defocusing, was discovered. Ergodic theory was created in the works of Boltzmann and Gibbs on statistical mechanics. My Doctor of Science dissertation was on applications of these new ideas in ergodic theory to statistical mechanics.

After getting my PhD from Moscow University, I didn't work a single day as a mathematician in the Soviet Union because of political reasons. This is why there was such a long gap between my two dissertations as well. I "was allowed" to defend my Doctor of Science dissertation after perestroika started and there was some kind of transition, but scientifically, there were no real changes besides a possibility to travel to scientific meetings abroad. I worked

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in many areas, but because of the strange, not usual way, I did not work as a mathematician after graduating from what was considered the best university in the world and defending an outstanding PhD. The same thing happened to many young mathematicians of Jewish origin. So I worked in many places. Maybe that is why my scientific interests and the questions I worked on are broad and perhaps even strangely broad.

I: You were actually applying a lot of mathematics to other problems after your PhD.

B: That is exactly what happened. I needed to work somewhere and couldn't find a job, not only me. The political situation was very bad and I believe that the disintegration of the Soviet Union started at that time; it was just concluded in 1991. I had to support my family, I was already married. So I found a job – it was 32nd place where I applied. It was at the Institute of Psychiatry of the Academy of Medical Sciences. They wanted to hire a mathematician because the institute had a computer. Two years later, when the computer broke down, they asked me to repair it. They thought that since it was a mathematical machine, mathematicians must be able to repair it. By then I had done some work in psychiatry and genetics. They realized that I could do something there besides repairing computers. So they hired an engineer to repair it, and I retained and continued my job there.

I: It seems that your scientific journey from Moscow University to Georgia Institute of Technology has been a very long one. Was it a planned one or did it just happen?

B: It was not planned at all. It was life with all its turns and changes, so on and so forth. For instance, my first job was in psychiatry and I applied mathematics to real problems in psychiatry and genetics. It is again the history of the Soviet Union. I was the only mathematician speaking at the First All-Union Conference of Medical Genetics. In Stalin's time, genetics was considered as capitalist science; it was forbidden, and many researchers working in this area were sent to camps. In high school we never studied genetics. I learned it only after coming to the Institute of Psychiatry and was fascinated by it. There were already new developments. Geneticists who had spent some time in prisons and camps and survived there came back and were working again on genetics, which again became an "allowed" science. They were much older than me. I was then under 30. When there were PhD defenses, it was very interesting for me. There were two people needed to read the thesis – the first was usually very senior, and the second sometimes was me if mathematics was used in the thesis. After the defense, it was the Russian tradition to have a banquet. That was where I learned a lot of real history of Soviet genetics. I wasn't doing mathematics, but I was doing an exciting science, and many people suffered much more than me. I couldn't complain.

I: Is the banquet after the defense organized by the department?

B: It's usually organized by the person who made the defense. It was just a traditional celebration. It is paid for by himself, but more often by his parents. It was the tradition that parents were very happy to have educated kids and sacrificed a lot.

I: Your research interests are wide ranging. Do you think that the Russian system of education has something to do with the range of your research interests and inclination?

B: It depends. In my case, yes. As everybody knows, Kolmogorov was one of the greatest mathematicians of the last century. Take his work on turbulence – it's the basis of turbulence theory for physicists. He was a mathematician, but he laid the foundations of modern turbulence in a 4-page paper. This is the style that I always admire. New clear ideas lead to some clear implications for real world problems. The longest time that I worked in the Soviet Union was in the Institute of Oceanology. This is, in fact, my third trip to Singapore. The first time that I came here was as a sailor essentially. Twenty-six years ago, I came with a scientific ship. I came here the second time, two years ago, and the changes in Singapore were very impressive.

I: Did you do any kind of experiments on board the ship?

B: I was a theoretician there, but almost everybody was an experimentalist. It was a long journey, about 4 months. I was young and strong and could help to carry heavy instruments, not just do theory. Most of the journey was devoted to the experimental studies of the oceans. On the way back, it was another thing. I had only a short time to somehow think over the results and to come up with simple models that would show that the results of measurements were correct and novel. I learned also to value the work of the experimentalists and how to talk to them, although they talked a different "language". Kolmogorov had also been on such a journey earlier on. He was there when his theory was under investigation. He really wanted people to check his theory with experiments. Then he published another paper which took into account the measurements. Of course, it was his influence. It was the style of his school. For example, when I was an undergraduate, my supervisor Professor Sinai would tell me about some dynamical system and said, "This is an interesting system. Look into that." But he did not always tell me what I should prove. By the way, some of the problems Sinai brought up came out of the research of one of the organizers of this program, Professor Zaslavsky, who was a physicist at Novosibirsk at that time.

I: The Russian tradition seems to be that theoreticians, even pure mathematicians, have a close interest in data and experiments.

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B: It's actually not like that. A great majority of pure mathematicians, because of the same reasons as mine, did not work as mathematicians but worked in some applied institutions. They naturally didn't like that and were not much interested in what was going on around them. Most of them were really pure mathematicians and didn't really understand what the physicists and engineers were talking about. They don't give definitions. It's a kind of personal thing whether you are willing to understand the things which are not exactly defined. For me, a formulation of a new mathematical problem and proof of that for some natural (hopefully visual as well) examples is much more important and exciting than trying to prove this for more and more and more general classes of systems. More and more technical ideas are needed for that, often combinations of various techniques, but I always prefer simple proofs. Now, sometimes a mathematical community gets embarrassed if a simple proof is found for a long-standing problem, whereas it should be to the contrary.

I: Would you consider yourself to be some kind of theoretical physicist?

B: In fact, some of my friends and colleagues call me a physicist. I don't think there is a big difference. I don't have a broad training and background in physics though it was part of our education. But I'm really interested in physical problems. It also depends on your scientific taste. Many of my results are just examples, and you can build a lot of generalizations on them. I'm more interested in the phenomena, maybe it's a more physical approach. I think all this is science. What I don't like in the US, for instance, is that they always say "Mathematics and Science". What is really part of the Russian mathematical system is that there are no sharp borders between scientific disciplines. If you remember, Francis Bacon, founder of natural philosophy, said, "Any science reaches a really high level only when it manages to use mathematics."

I: Is chaotic dynamics a recent development of the chaos theory of the seventies?

B: "Chaos" is (actually was for a long time) a very good selling word. Chaos is just a part of that new branch of science which Kolmogorov founded in 1958, twenty years before the word "chaos" was coined. It was called stochasticity of dynamical systems, which means that dynamical systems, purely deterministic systems, can demonstrate the same behavior as purely random systems. It was a real physical and even philosophical discovery. To the general public or people who give funds, what is "stochasticity of dynamical systems"? It is something vague. So "chaos" was coined and chaos is only part of this general area of stochasticity of dynamical systems. Chaotic dynamics is just one face of complex dynamics. The first book on this subject was published by George Zaslavsky in 1970 under the title "Stochasticity of Dynamical Systems". But "Chaos" completely took over. You know, when something becomes more fashionable, you give up something else.

I: The term "chaotic dynamics" seems to suggest more of a physics discipline.

B: It's not only a branch of physics. It's a branch of science – it's also chemistry, biology, geology, geophysics and many other disciplines. But, of course, first of all, physics. Physicists are mathematically trained and they can use the computer better than mathematicians. This is why it was first used in physics. There are many physical systems that develop chaotic behavior. What is the basis of that? Historically, what people knew for centuries, starting with Laplace and even before, is that if you knew exactly the initial conditions of your equations, and the functions involved are sufficiently smooth, then there is a unique solution that can completely predict the evolution of the system. But in practice, if you have any measuring device like a thermometer in physics or medicine, you never have complete precision. You know approximately how you drive your car, approximately 50 mph, maybe 51, but not 51.603. In any practical situation, you work with some such small set of data, not a point. You study the evolution of this small set, and very often it does not look like the evolution of points. This is where all the chaotic dynamics occurs. You have very good precision at the beginning, but with time you lose it. Your prediction can only be statistical.

I: What are some of the central problems and recent advances of chaotic dynamics?

B: This is a kind of natural evolution and development. The major discoveries were in the late 50s and 60s by Kolmogorov, Sinai, Smale, Arnold, Moser, Anosov. Dynamical systems evolution can be very complicated. Another of Kolmogorov's work said that not only a dynamical system's evolution can be complex, but the simplest (integrable) dynamics in Hamiltonian systems is actually stable. It is the celebrated Kolmogorov-Arnold-Moser (KAM-) theory. In practice, you see all those things that are stable under small perturbations. There was a general belief though in physics that if you take a surface (manifold) of constant energy, then the motion is ergodic there, uniformly distributed, but KAM-theory said that it's the opposite situation if you have integrability. Integrability is stable, chaoticity is stable as well. The studies so far took care of these two polar situations – complete chaos and near integrability. The most challenging problem now is: what is in between? The system is neither integrable nor chaotic. Instead, it has a mixed behavior – sometimes it's divided phase space – you have islands of stability in phase space that are called KAM- islands, situated in a chaotic sea. It is much more difficult to study such intermediate

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systems. Some methods were developed to study the chaotic dynamics, another totally different methods were developed to study stable dynamics. But, at the border of these islands in chaotic seas, you cannot apply any of these methods. This is the major challenge and development.

Dynamical systems behave like stochastic processes. First of all, we are looking at the simplest random processes – coin tossing, independent random variables. But most often, in applications you don't see independent random variables, you don't see Markov processes, but you see processes with infinite but decaying memory. This is a much more difficult problem. There were some breakthroughs. For instance, the simplest examples were found which demonstrate the co-existence of islands and chaotic dynamical systems. By studying these examples, one can completely analyze the system and generalize the theory. The basis of the theory of dynamical systems and ergodic theory often comes from some simple classical examples. One of the major efforts now is the study of the so-called Arnold diffusion – if you start in the chaotic region, how does the particle move between the islands? Does it move fast or slowly, can it move far and so on?

Another development is related to general questions in communications theory and biology, and it has to do with interacting dynamical systems, like systems of neurons, communication networks. We now know rather well how finite-dimensional dynamical systems may behave. But suppose you have several such systems that are connected. Then some new general questions appear. How does the whole system behave? It's space-time dynamics. Not only dynamics in time, but in space because you have different local systems (elements of a network). How do networks behave? It raises questions about different types of synchronization, space-time chaos, etc.

I: Are there are general results for such questions?

B: There are very few results so far; only for some special classes of dynamical networks. But there is no general theory. This is a major challenge.

I: Is there any theory for infinite-dimensional dynamical systems?

B: Again for some classes, such a theory exists, but usually it is not something which is likely to have real applications. But it is very important to find a class (even a narrow one) of systems where we can understand everything. It helps to build intuition on what to expect in the evolution of more general networks.

I: You did some work on chaotic motion of billiards. Is it related to the Hadamard billiards introduced more than one century ago?

B: Hadamard was one of the pioneers in studies of chaotic dynamics. What is now called "Hadamard billiards" is not really billiards. What "billiards" means is that you study the motion of a point particle, mechanical particle, or an acoustic wave propagating in some medium. It gets reflected from the boundary. If there is no boundary, it is not a billiard. Systems without boundaries are the simplest – they are just geodesic flows and were studied before billiards. Hadamard's fundamental work is not really about billiards. I was surprised to hear this name "Hadamard billiards". It was given by a physicist working in chaos theory. It's kind of confusing, but it's in the literature now. Unfortunately there is much confusion in giving names in chaos theory. Many people are working with billiards in applications because it is a very natural physical model in mechanics, in statistical physics as well as for light and sound propagation, in mesoscopic and in atomic physics.

I: What about some of the advances in percolation theory?

B: In percolation theory, I was only working tangentially and would be embarrassed to talk about it as I'm not an expert.

I: Are there any surprising or counter-intuitive discoveries in your research work?

B: There were quite a few. The first was right after my PhD – there I proved a theorem for a rather general class of systems of billiards. But after it was published, I realized it had some consequence which was very counter-intuitive. I published a very short paper, which is a special case of the research conducted in my PhD, and this paper had a hundred times more citations than the general paper. This result is very easy to explain.

Consider a narrow parallel beam of rays emitted by some flash light. Let this beam propagate in two-dimensional planar region (a billiard table) with mirror walls. It gets reflected from the mirrors. Question is whether the entire region will be illuminated – that is, for all points inside the region, some ray will pass through them. If all mirrors are convex inwards – this was introduced by Sinai – the beam becomes broader and illuminates much more. But if it is a concave mirror, like a circle, it illuminates less. Therefore there was a universal understanding that if you have dispersion at the boundary, then it is strongly chaotic – it illuminates everything and you lose precision fast. In a circle, the beam of rays just goes around and there is no illumination of the central part of the circle. I considered a perturbation of the dispersing boundary by small focusing pieces; then it will still be chaotic. It doesn't sound surprising. It occurred as though there is another mechanism of chaos generated by the focusing boundary.

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For instance, if you take a circle, cut out a small piece by a chord and consider a billiard inside such table, then it is strongly chaotic because of defocusing: between any two consecutive reflections a beam of rays will pass through a focusing point and become divergent, like in billiards with dispersing boundary. Eventually it will illuminate the entire region. This was a real discovery – nobody thought about that, I didn't expect it either. The funniest thing is that everybody refers to this short paper rather than to the one from which it follows. This mechanism of defocusing was found in many other systems. It revealed that chaotic behavior is much broader phenomena than people thought before.

I: Are there some strange physical consequences of that kind of behavior?

B: There are; actually, experimental physicists in many physical labs constructed this type of devices and studied this phenomenon. This is what I like. When you come up with some clear examples, physicists go to their labs and build real physical analogs of these "purely mathematical" toy models.

Another example is from a totally different area, in psychiatry. My first papers were published in genetics journals. At that time, there was a theory by an outstanding geneticist in the Soviet Union that any hereditary disease is confined to genetic families (consisting of all ancestors); that is, roughly speaking, if two persons are carriers of a hereditary disease, then there is a high probability that they are relatives (in the genealogical tree). There was such a strong claim based on some computations in genetic populations. It is a fundamental problem for the organization of health care. Dealing with it, I introduced a new class of models in population genetics, which was called hierarchial models of human population. Models that were considered before assumed that the population is mixed, people get married randomly or there are several such populations with (horizontal) migrations between them. But we see that people from small villages usually migrate to cities, from small cities to bigger cities, and so on. Migrations in the opposite directions are essentially negligible. Of course, there are not so many layers, roughly speaking four or five even in the developed countries. However, this hierarchial structure is very important and changes the behavior of the population very essentially. My computations for such hierarchial populations gave the distribution of the special genes that are carriers of hereditary diseases.

Several years later I was at a conference in mathematical physics, and a physicist from Germany asked me whether I had a brother. "Yes, I have a brother," I said. "Oh, your brother is working in population genetics," he said. "No, it's me, not my brother". He was very surprised, and said that he had a friend who worked in demography, conducted experimental studies in Germany and could not explain the results and measurements, especially in urban areas. Then somebody told him about my model and everything was explained. It was exactly the same situation – there are many relatively big cities close to each other, coal miners were living in Essen, Dortmund, Duisburg, etc. Thus several big and well-developed cities are extremely close to each other, and form the rich high level in the hierarchy of migrations. The demographers there said the population structure did not fit any model, but the hierarchial one worked quite well.

I: You were already working on problems in biology and medical science long before the Human Genome Project. Have you applied your ideas to bioinformatics?

B: Actually I'm working in bioinformatics in Georgia Tech. We have a big effort in bioinformatics there. In Georgia Tech we had the first Master of Science program in bioinformatics in USA. Now we have also a PhD program in bioinformatics. You know, bioinformatics is another buzz word in a sense. I like it; it's better than chaos. But still, some people ask, what is bioinformatics? To me, it is analysis of medical and biological information in a general sense. But often people refer to it merely as the computer analysis of long molecules like in the Human Genome Project – DNA, proteins. A few years after the Human Genome Project, we know the letters but not the language; you don't know what is written by these letters. I think it's extremely tempting to bring in mathematics at this level and this is what people are trying to do. I believe that one of the major problems with biology is that there are no biologists who, like physicists, know and understand mathematics. All areas of mathematics are based on calculus or analysis. Historically, all the examples there were taken from mechanics and physics. Biology majors are not interested in calculus courses because there are no examples from their science. This is one of the major obstacles we need to overcome and this is what we are doing in Georgia Tech. We have developed new courses and now have several sections of calculus: traditional for engineering students and a new one for life sciences students. It's not a big deal. We just collect examples from biology, chemistry, biochemistry, genetics as a basis of this course. I hope that in 5 years' or 10 years' time, a new generation of biologists educated in mathematics will appear. A new thinking is needed.

I: Biology is changing very fast nowadays.

B: Yes, but still very slowly. Computers are now used and many people believe they can compute everything. But you should understand what you have computed. Here mathematical modeling is necessary.

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I: How much of the computer do you use?

B: I don't use it myself but I really appreciate this possibility to conduct mathematical experiments. It's great. You have some idea and you can see whether it works or not by simulations. My students and collaborators use them.

I: Do you have a lot of graduate students?

B: At Georgia Tech, I usually have 3 graduate students. On the average, in our department, there is one graduate student per faculty member. I don't know whether 3 is a lot. In some other places, people have more.

I: Can you tell us something about the Southeast Applied Analysis Center?

B: Actually, it doesn't exist anymore. It was created by the Georgia Tech Department of Mathematics which became one of the leading research departments. We won a tough competition for a NSF grant with other departments in US. We were running projects and lectures for a lot of universities and colleges in the Southeast informing them about new developments in mathematics. We also had postdocs and some of them became visible researchers and won prestigious prizes. We are now trying to launch another center which will be more oriented to biology and ecology. The Southeast Applied Analysis Center was more oriented to probability and discrete mathematics. There are no more funds for this program now. In US, if there are no funds, it is just a name. So SAAC naturally disappeared.

I: The new center you mentioned is a kind of successor?

B: Yes, it is a successor. It is a kind of natural and major development for Georgia Tech where biological studies became a high priority area.

I: What will the new center be called?

B: I suggested "ABC Mathematical Center". A stands for "applied", B for "biological" and C for "contemporary mathematics" – contemporary in the sense that ABC will be more oriented to the studies of new contemporary topics like biological networks, systems biology, evolution biology, cell biology, bioinformatics, infectious diseases and ecology.