Persi Diaconis : The Lure of Magic and Mathematics >>>

An interview of Persi Diaconis by Y.K. Leong

Persi Diaconis is perhaps one of the most unusual mathematicians of our time. After studying the violin when young, he switched at the age of 14 to magic in which he had a successful and colorful career for almost ten years, and then, at the age of 24, he made another decisive switch to mathematics. He has made numerous contributions to mathematics, statistics and probability and is editor of many well-known journals. He has also used his expertise in mathematics and magic to investigate claims in parapsychology. He has been an invited lecturer at important meetings, notably as Wald Lecturer of the Institute of Mathematical Statistics, Gibbs Lecturer of the American Mathematical Society, plenary speaker at the International Congress of Mathematicians and Von Neumann Lecturer of the Society of Industrial and Applied Mathematics. He has won many prestigious awards and honors, and is a Fellow of the American Academy of Arts and Sciences and a Member of the National Academy of Sciences (USA). He is now Mary Sunseri Professor of Mathematics at Stanford University and holds joint positions at the Mathematics Department and the Statistics Department.

The Editor of *Imprints* interviewed Persi Diaconis on 20 August 2003 at the Institute for Mathematical Sciences when he was at the Institute as an invited speaker for the program "Stein's Method and Applications: a program in honor of Charles Stein" held from 28 July to 31 August 2003. In the following edited transcript of the interview, he talks about his two loves (magic and mathematics) and the excitement of research.

It's mathematics, not magic

I: Thank you very much for agreeing to be interviewed. You started to take up mathematics at a comparatively late stage of your life. What made you make that change when you already had a successful career as a magician?

D: At 24 years old. I really don't know why I went into mathematics. It seemed to be an esoteric subject to me when I was young. It was the Sputnik era – if you get a PhD you could get a good job as a professor (it's still the case, I think). Maybe that's the reason.

I: There must be some connection between what you did before and what you did after.

D: There was some connection in the sense that I knew Martin Gardner who was a wonderful writer of popular mathematics. He put some of my early magic tricks that were mathematical into the Scientific American and I was very happy about that. I didn't really know any other mathematician, but in the end I felt there was some similarity between mathematics and magic.

I: You already used mathematics in your magic tricks?

D: Oh yes, there are some magic tricks that use pretty elaborate mathematics. For example, magicians can perfectly shuffle a deck of cards. I learnt that if you do that eight times, the deck comes back to where it started. One of my early discoveries involved the two types of perfect shuffles, in and out. The out shuffle leaves the original top card on top, the in shuffle brings it second. If you want the top card to be in some position j say, then you express $j - 1$ in binary form and use the bits of $j - 1$ as instructions for the shuffles. That's how you get the top card to where you want it to be. Well, that's a mathematical discovery. There are all kinds of tricks that use mathematics, most of them awful, but there are some good ones.

I: So you actually discovered some mathematics while you were in magic.

D: Something about binary numbers, also Fermat's Little Theorem $2^{p-1} \equiv 1 \pmod{p}$, and some elementary number theory.

I: You mentioned before that doing mathematics is like doing magic. How is that?

D: One similarity is this: you have to solve a problem and you have certain tools that you are able to use and others that you are not allowed to use. And as in problem-solving there is the notion of elegance. The difference is that mathematicians have hundreds of years of tools whereas in magic you use whatever you can get. The similarity is especially so in applied mathematics in which the problem

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comes from somebody else. The chemist or biologist might have a question for you, and you don't have any ready tools. You have to start thinking about it and start using whatever tools you have or invent new ones. That's pretty similar to solving magic problems.

I: In some sense, magic is as logical as mathematics.

D: In some way, but magic is not logical enough to allow everybody to see the trick. If you think about it, magic involves principles of deception. I once taught a course on the history of deception. It was an interesting course about the history of magic and the psychology of lying and things like that.

I: Did your experience with magic give you some advantage when you took up mathematics?

D: The only advantage would be first of all that I had the ideal of inventing something of my own from my magic tricks. Also I had a great mentor (Dai Vernon) who taught me the difference between mere variation and something that is really original. Another thing is speaking before the public. It makes a difference to be able to give a talk to people. To make them understand a talk is like doing a show. You have to make them follow it and enjoy it and not just sit there thinking how smart the guy is.

I: When you prove a theorem do you try to find the best way to do it?

D: Sometimes. Erdös once had a theorem about the order of a random permutation. He and Renyi had a very long paper on the distribution of the order of a random permutation. I managed to prove it in about five or six pages, and I showed it to him and he looked at me, surprised that I was right. And I remember that.

How do you know that something that you use a lot is true? I'm opposed to the recent thing about proofs from "The Book". They are wonderful, amazing proofs but they are useless. They are not the work of mathematicians because that's not the way we work. You want to tell people how you think about your problems.

If you can follow the proof why that's true, that's great. But these very sleek proofs … are only beautiful like magic tricks

I: You don't go back to re-prove things?

D: Sometimes, but not often.

I: Do you choose to work on a problem because of its potential applicability?

D: Not necessarily. I'm always happy if a problem has an application. I used to just make up the problems on my own, or work on a simpler version of a real problem if it's too hard now. I work on mathematical problems with biologists and problems from computational group theory which involve computing with large finite groups. One of the main tools in group theory is the representation theory of finite groups. I have used this extensively but also contributed by working out a non-commutative Fast Fourier Transform.

I was recently working with group theorists to try to understand why the product replacement theorem works. One of the ingredients is that you have two giant matrices, say 100 by 100, which generate a group over a finite field. You want to know something about its irreducible representations. You take a random element from the group algebra and do something like a random walk on the matrices, and it works pretty well. You can prove a theorem about it. Well, I asked myself what can you say about the general linear group over the integers modulo 2. The question led to a simple urn problem. It was an urn problem which I could not do until I came here and discussed it with Zhidong Bai and we could do it. I was working on the general linear group. It was too hard but I looked at a simpler problem and we solved it. I'm working on problems in group theory by applying probability. You have to know a lot of group theory and vice versa. I'm fascinated by problems from group theory.

I: Is this a new approach to group theory – applying probability?

D: No, not really. Many people have used this kind of approach. Erdös has used probabilistic methods to prove that an element of a group has certain properties. If you want to know whether a group is simple or whether a certain modular representation is irreducible, there are algorithms which can be used to show that they have the required properties with probability close to one. But it raises a philosophical question: if something is true with probability $1 – (1/2^{100})$, is it the same as saying it is really true?

I: How important is collaboration with people in other fields?

D: I learn many things in that way – things that I don't know about. It's very hard to read something from the papers in a different area. But we can talk to each other and in ten minutes you know what it is about. That's one of the great joys in mathematics – to get to talk to somebody.

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There is something I once learned from Erdös very early on. We were working on a number theory problem. It came down to an algebraic topology question, and he said, "Oh, I'll just call so and so, and I'll ask him." And I was shocked because I thought that we were learning from the topology books and shouldn't we work it out ourselves, but he said, "But why? If we call him, in five minutes we'll get the answers." I thought that if we want to learn some algebraic topology, we would be better off learning on our own. Anyway, there's the question of when do you ask somebody else and when do you learn it yourself?

I: How do you describe yourself with respect to your research?

D: When people ask me what I am, I say I am a statistician. I certainly do a lot of probability, but I never had a proper course in probability. I taught many courses in probability and I worked with great probabilists who taught me. When I was at Harvard, which is a very mathematical department, we didn't really have a probability course. There was a course I took on probability given by Gleason who would just teach it without knowing the literature. And he taught it in an original way. Then I try to learn group theory, and now I know more group theory than most probabilists.

I: What is the most satisfying piece of research work that you have ever done?

D: I would say the work on perfect shuffles and random permutations.

I: What are your effective working habits?

D: Well, they have changed. It used to be that I would wake up early in the morning and work till late at night. And then ten years ago, I met Susan. Now I wake up early before the others and work for a little while and then I have to find a place to hide. If I go to the office, there are students, the email and the secretary. So I have to find some place to hide for an hour or two. My work is now more sporadic than it used to be, but I still work at night.

I did a lot of work with other people on the phone. I would call people on the phone and tell them I was stuck with this problem and asked them would they tell me about this and that. It's nice to talk to people.

I: Do you think about your problems when you are doing something else?

D: Sometimes. I love to solve problems, I love to think about them. Sometimes I just have to relax. It's a hard thing to do.

Some people ask me whether I work on big problems. I just work on the next problem in front of my nose. I work and think hard about it. When I came to Singapore, I started to think about new problems. Before that, I was thinking about some problems about symmetric operators commuting with groups of transformations.

I: How much of the computer do you use?

D: I used the computer a lot in my PhD thesis. I love to think about algorithms, about better ways of computing, but mostly I leave the computing to others.

I: Have there been any surprises in the way your research work has developed?

D: Sometimes. There have been wonderful surprises. To study the perfect shuffles used in magic tricks, you have to take two permutations and look at the order of the group they generate. It turns out that they are very useful for certain parallel processing algorithms. Well, some people don't like their work to be useful. But I wrote a book about representation theory of groups and probability theory and try to understand what it means for two matrix representations to be close together. I wrote a few papers about that and had some PhDs writing their theses on that too. It's wonderful for me to have a bunch of computer scientists reading my papers and asking technical questions which I considered twenty years ago. That's amazing and really surprising. Several times, I'm worked hard on a problem for my own reasons, and then somebody else is interested in it too. Some of my colleagues screen their problems. I'm just the opposite – though I do screen my problems. If I work on a problem and I have to talk to other people or to learn some new mathematics, that's a good reason for doing it.

I: Are there any problems you would like to solve?

D: I worked hard on Stein's Method when I came to Stanford. I thought I knew about it but I didn't do very much. Now I'm thinking about it again. In the airplane, I was thinking about eigenvalues. When I came here I worked hard with Zhidong Bai and we got some work done. I don't know what problems I would really like to solve. If you think about it, wouldn't it be wonderful to solve the Riemann Hypothesis? But I just don't work like that.

I: What about problems in computational biology?

D: I tried to teach myself biology for several years. It's quite embarrassing how little I managed to learn, to tell you the truth. When I first came to Harvard, there was a young assistant professor who took one of my courses and who was involved in the university's work on the human genome. He came to me with some combinatorial problems and he tried to teach me some biology. I wrote some papers on computational biology but only if somebody posed the problem as a mathematical problem. My wife Susan really learned the biology and all the other business. It's several years of work. When it came to learning biology or learning about another part of mathematics, like unbounded operators or extraspecial p-groups, I had to choose, and I chose extraspecial p-groups. It's some internal reason.

I: If a graduate student has to choose a field of research, what kind of advice would you give him?

D: I had a lot of graduate students coming to see me about research. After going through graduate school, they should know what they want to do for their PhD thesis. (I did my thesis on Tauberian theorems in number theory - but this is not what I do nowadays.) If a student has a topic, well, let's find some problem that is relatively hard to do. Another way is to take a harder field and learn about it, Sometimes I say to students "Do what you can do best." All students are different. I remember one student in my class who was spectacular with his homework problems. People were telling him " You're good. Why don't you read this." But he wouldn't sit down and read those things. He was a problem solver. So I said, "Okay, here are some problems. Solve them and that will be your PhD thesis."

I had another student who wanted to do something real. I didn't help him very much but I didn't get in his way. I didn't make him do work that I thought he should do. I let him do something real – character recognition to read Bengali. It's partly pattern recognition and it uses algebraic topology and curves, and it really works. I'm very impressed by that. My students are free to do all sorts of things.

I: There is a lot of pure mathematics being applied to statistics, isn't it? Like Jordan algebras.

D: Absolutely. Jordan algebras in multivariate analysis, symmetric function theory, Charles Stein's work on amenable groups. Statistics used to be more theoretical but now the computer has taken over. Statistics has become very computationally oriented. They are not interested in group theory. I'm half in the mathematics department and half in the statistics department. It's not clear what we are doing is relevant to statistics.

I: What do you see are the directions in statistics?

D: Well, very big data sets. For example, I'm working on this problem on protein folding with enormous data sets and the problem is how to simplify them and make them comprehensible – data mining. How do you adjust to the richness and the power of the computer? It's an important problem. The technology is changing rapidly. So I may not be the right person to ask.

I: You have been very enlightening on many aspects of research which we don't see from journals and books.

D: People don't talk about what it is really like to be doing mathematics. If you work in a new area like me, you go and talk like a child to somebody who has been doing it for twenty years. If you can tolerate feeling stupid, something will come out of it. When you can finally think your way out for five minutes, you will say, "It's nice, How can I have missed that?" I often think that most of us who keep going have pretty thick skins.

I'm very happy that you have the IMS. It's something important to have and it's a wonderful idea. You don't have anything like this in the area. It's great for the country.

I: But we are a bit isolated. We are grateful that people do come. It's a long way away for many of them.

D: Well, you can bring people in. If you put up some good meetings and you do as well as you have done, people will come to this place. You gain some reputation for good work and it has been a wonderful conference to me. I feel really invigorated and I learn something I didn't know about new problems. Some of the youngsters are very good and I've been working hard. Everyone is working hard. It's a pleasure working. We don't know ourselves. We don't know how happy we are to be allowed to do what we do.

I: Thank you for your kind words. It's encouraging for us at the Institute.