Roger Howe: Exceptional Lie Group Theorist >>>

Roger Howe

Interview of Roger Howe by Y.K. Leong (matlyk@nus.edu.sg)

Roger Howe is well-known for his path-breaking work in the theory of Lie groups and representations and for his impact on mathematical education and pedagogy through his teaching, writings and active involvement in educational reforms. His research is also directed toward the applications of symmetry to harmonic analysis, group representations, automorphic forms and invariant theory.

He has a bachelor's degree from Harvard University and a doctorate from the University of California at Berkeley. He taught briefly at the State University of New York at Stony Brook and, since 1974, he has been at Yale University where he has served as director of graduate studies in the Department of Mathematics and as departmental chair. He has held positions at the Institute of Advanced Study in Princeton, University of Bonn, Ecole Normale des Jeunes Filles in Paris, Oxford University and Rutgers University, Institute for Advanced Studies at Hebrew University in Jerusalem, University of Sydney, University of New South Wales, University of Metz, University of Paris VII, University of Basel, Kyoto University, National University of Singapore, Hong Kong University of Science and Technology. He has

Continued on page 14

Continued from page 13

been invited to lecture in many countries throughout the world. He is a member of the National Academy of Science (US), the American Academy of Arts and Sciences and the Connecticut Academy of Science and Engineering. He received the American Mathematical Association's Lester R. Ford Award for exposition. He is an exceptional research mathematician who also serves selflessly and tirelessly on national and international boards and committees for the advancement of mathematics and the improvement of mathematical teaching and education. Among others, he has been involved in the CBMS project on the mathematical education of teachers, AMS Review Group for revision of the NCTM Standards, NRC Mathematics Learning Study and AMS Committee on Education. He was on the Board of Directors of the Connecticut Academy for Education in Mathematics, Science and Technology and the Mathematical Sciences Education Board at the NRC. Recently, he received the 2006 American Mathematical Society Award for Distinguished Public Service.

In recognition of his distinguished scholarship and exceptional teaching, he became the first incumbent of the Frederick Phineas Rose Professorship in Mathematics, and he was recently appointed the William R. Kenan Jr. Professor of Mathematics at Yale. His influence on his students is well recognized. In particular, his influence is strongly felt in Singapore in his role as chair of the Scientific Advisory Board since the establishment of IMS in 2000. He has also bequeathed part of his mathematical legacy to the Department of Mathematics in NUS in the form of a strong research group centered round a number of his returned PhD students. In his honor and in appreciation of his numerous contributions, an international conference was organized at NUS from 6 to 11 January 2006 on the occasion of his 60th birthday. When he was here as the guest of honor of this conference and also for the annual visit of the advisory board, Y.K. Leong interviewed him on behalf of *Imprints* on 7 January 2006 at a café near Swissôtel The Stamford, Singapore. The cacophony of the surroundings and the cold from which he was then recovering did not dampen the passionate spirit with which he talked about mathematics. The following is an edited and enhanced version of the transcript of the interview.

Imprints: You did your B.A. in Harvard. What attracted you to Berkeley for your PhD?

Roger Howe: The main factor was that it was in California. I had spent my high school years in California and I still considered myself a Californian. Although I was in the east for 4 years, I really wanted to go back. Berkeley was the best-known place in California while I was there. In some sense, I was naïve. I didn't even think about Princeton. I didn't know that Princeton was the place you might want to go to. In some sense I should have stayed in Harvard because I had won the Putnam Competition and that included a fellowship to study at Harvard. I think that some of the faculty there were somewhat shocked that I decided to leave, but I was very intent on getting back to California at that time. A more substantial reason for going to Berkeley was that it had both a very large and very strong faculty so that you can study almost anything.

I: Was anybody there whom you particularly liked to work with?

H: I had already gotten interested in representation theory, in which I have done most of my work. George Mackey who was at Harvard (I did a reading course with him in my senior year) had a student there [at Berkeley], Calvin Moore. He ended up being my advisor.

I: You mentioned the Putnam Competition. When you went to Berkeley, did you have a scholarship or something?

H: Yes, they actually offered me a pretty nice fellowship. There were some special fellowships from the government intended to recruit students into scientific areas and I got one of those fellowships. It was called an NDEA (National Defense Education Act) Fellowship. The NDEA was motivated by a desire to keep ahead of the Soviet Union in science. It supported many graduate students who went on to productive careers in mathematics and science.

I: Why did you choose to pursue research in "pure" mathematics?

H: Well, actually I hesitated a lot. It bothered me that there seems not to be more emphasis on connections. To me the applications of mathematics are a very appealing part of the subject and it is very important to me that mathematics helps you understand the world, but eventually I ended up going into pure mathematics and I have been very happy working there.

I: No regrets?

H: Not really, no. Well, yes, some; I wish I had done some more applied kind of things but it hasn't worked out.

I: Do you consider yourself to be an analyst first and then an algebraist?

H: Well, this is going to sound kind of funny but I actually consider myself to be a geometer. I kind of think geometrically. I've never published any work specifically on geometry but I love the subject very much. Euclidean geometry was one of my favorite subjects in high school and I've continued to think about it. I'm working with a

Continued on page 15

Continued from page 14

colleague on a textbook for geometry. I think in terms of pictures although my work doesn't seem to have much to do with geometry.

I: Not many mathematicians think geometrically, they think more symbolically.

H: I think topologists often think in terms of pictures. Algebraists and analysts probably have other means of figuring things out. For some topologists at least, pictures are very important.

I: Your work is connected to some kind of topology, isn't it?

H: Lie theory actually – this is an attractive aspect of it to me – it connects with all branches of mathematics. It is geometrical but it is also algebraic, it is also analytic. Some of the main examples of geometric figures of manifolds and constructions come from Lie theory. That's a very attractive part of the field to me. You can connect things.

I: Are you a theory builder or a problem solver?

H: I like problems. I often will work on problems but I have never published a paper that solved a specific problem. And I very much like making connections, sort of coordinating things and connecting things together. On the whole, I guess that I'm a theory builder.

I: To be a theory builder, one has to know a fair number of fields to see the connections.

H: It helps to know different subjects. I have to say that I'm quite surprised how useful many things I learned for no particular reason have turned out to be… you learn things that don't seem to have connections with one another, and later on, you do find that there is some way to relate them together. That's very satisfying.

I: Algebra, or for that matter analysis, has its origins in rather "concrete" problems but modern research in these areas seems to be getting more and more abstruse and esoteric. Do you think that this is a desirable trend?

H: I think there's a constant kind of dynamical dialectic between what seems to be very abstract and the more concrete things. A very dramatic example in recent years is Mandelbrot's exploitation of fractal geometry. The basic work that Mandelbrot has exploited or publicized was actually invented by mathematicians in the late 19th century and early 20th century, and they were for a long time considered to be extremely abstruse constructions that could never have anything to do with reality. These were prime examples of things that only mathematicians would ever think of. And then Mandelbrot came along and said, "Actually, clouds, coastlines and trees and many, many different shapes in nature share some of the qualities of these structures, and we can learn about nature by thinking about these seemingly very weird structures." There's always a pull between the abstract and the general and things that seem to be far from reality, on one hand, and very concrete things on the other. It's also the case that physics is very weird and physics had to go far beyond our basic intuition in order to uncover a more fundamental truth in nature. I think mathematics is like that, and of course, mathematics is a major tool in physics.

I: Did Mandelbrot discover those things independently or did he already know about them?

H: He was aware of the earlier constructions. He was the one who was able to see that there are things in nature that are like fractals.

I: It seems that the success of algebra is its ability to reduce problems to symbolic manipulation but that the ideas of analysis are more intuitive and their formulation often precede their justification. Do you agree with this view?

H: Of course, there are some famous examples of that, like the Dirichlet Principle which was used in the 19th century for a long time before it was justified. This may be an example of mathematical riches in a direction that we don't very well understand but when we come to understand them we can reduce them to more basic things. I also want to say that in Lie theory there is a very interesting dynamical interaction between the algebraic and the analytic. It turns out, for example, that a fairly important aspect of representations of Lie groups is that there are some natural functions associated with them called matrix coefficients. An important fact is that for many groups, matrix coefficients die off – they go to zero at infinity, and this has implications for ergodic theory and counting rational points on various varieties. The proof of that is a very interesting interaction between the algebraic structure of the group and the analytic structure of some vector space. Again and again in Lie theory, you find these things, which you think of as different, interacting in an interesting way.

I: In general, geometric intuition seems to be very nebulous and often initially the ideas do not have rigorous justification.

H: It's hard to pin down, yes, but then you can spend very profitable, maybe very long, periods trying to figure out what are the reasons why this thing is true and you learn a lot during that process.

Continued on page 16

Continued from page 15

I: It's interesting what you said earlier on that you think in terms of pictures. That means that your intuition is basically a kind of geometric intuition.

H: I think it is.

I: It's quite rare, at least among algebraists.

H: It's hard for me to say. You can only know how you think. You can't know how other people think.

I: Could you briefly tell us some of the central problems in your area of research?

H: In representation theory in the strict sense, I guess the major fundamental problem still open is the classification of unitary representations. This has proved to be a very hard problem, Interestingly, the collection of all representations of a reasonable form has been known for 20 or more years but to figure which ones inside that set are unitary turns out to be a very hard problem. Then there are applications of representation theory to the theory of automorphic forms, and there, there are a huge number of problems which prove to be extremely challenging. A large part of it is what is called the "Langlands program" which has been challenging many mathematicians for several decades.

I: Has there been much progress in the Langlands program?

H: There has been very substantial progress but I'm not the best person to comment on it. In particular, Jim Arthur has established a rather general version of the trace formula and he has made applications of it. That's a good example where there is intuition and things are not proved, so there is a large web of conjectures. Only people who spend all their time thinking about them have a clear picture of what part is known and what part is conjecture. It's quite an amazing zoo.

I: How does it compare with the classification of the finite simple groups two or three decades ago? There were then a lot of things floating around.

H: That was a fairly well-defined problem. Of course, before the classification was achieved, people didn't know how far one would have to go. But the Langlands program is much more open-ended. I think that it includes problems that we, in fact, will never solve.

I: You mentioned the unitary representations. What is the significance of unitary representations?

H: Well, in physics, that is, quantum mechanics, representations arise because of symmetries of a physical system. And the representation should be unitary, because the states of the system come from vectors in a Hilbert space, and the inner product has a physical interpretation. In the theory of automorphic forms, again there is a natural inner product which was constructed in special cases before it was realized that representation theory was relevant. And of course, unitary representations have a nicer theory than more general ones, just as statisticians liked to use least squares approximation, because it is nice mathematically. However, not all representations useful in applications are unitary.

I: What are the prospects of settling this problem within the next 10 years?

H: Very good progress has been made. In fact, it is now understood that for any given group the problem of classifying the unitary representations comes down to a finite algorithm, but the question is: can it be made more specific to form some kind of global picture? Also, there are computational issues because for some of the exceptional Lie groups the computations that you have to do to carry out this algorithm may be very, very expensive. It's not a problem about which you know nothing. A lot is known. David Vogan and Dan Barbasch, in particular, have made a tremendous amount of progress, but it's not settled yet.

I: Has the computer been used?

H: Computers are being used. In fact, there is a group now working on setting up a website where you can go and input a given representation of a given group and it will compute for you whether that is unitary or not.

I: From your personal point of view, you would prefer something more conceptual?

H: More pictorial, yes. We need a more geometric picture of it.

I: What are some of the recent applications of your field to other areas?

H: The subject of matrix coefficients has applications to ergodic theory and counting of points on varieties (equidistribution of points in some larger space). That is one kind of applications. Of course, there is the ongoing application to to the Langlands program, the theory of automorphic forms, where there is a constant interplay between representation theory and a broader spectrum of number theory. Recently, S. Alesker has used group representations to settle some outstanding conjectures in geometric integration theory.

Continued on page 17

Continued from page 16

I: What about to physics?

H: Well, this is what got people interested in representation theory at the start. Some representations of certain groups, you could say, are in some sense at the center of mathematics and at the center of the universe; in particular, a relatively simple kind of group called the Heisenberg group. Mathematicians call it the Heisenberg group because it is the group-theoretic embodiment of the Heisenberg canonical commutation relations in quantum mechanics. A tremendous amount of mathematics and physics relate to that group. Differential equations come in naturally and there are several absolutely fundamental mathematical structures which are connected with that group. There's the basic result of Hilbert's syzygy theorem in invariant theory or the linear algebra behind the Hodge decomposition of cohomology on Kaehler manifolds. So much is connected with this particular group that it's really amazing. Also, the quantum-mechanical hydrogen atom which is the basis of our understanding of chemistry is essentially a certain extremely distinguished group representation. So group representation theory in some sense is fundamental for our understanding of quantum mechanics. But it connects to many other things too. The odd thing is that these physical systems carry extremely special, very interesting representations of certain groups and the challenge is to find out what are the uses of more general representations. There have not been that many applications of general representation theory as we would like to have.

I: There are those super-Lie groups. Are they generalizations of the standard ones?

H: Yes, they are sort of combinations of several algebraic structures in one. Lie algebras are based on a product which is skew-symmetric – if you switch the order of two elements in a product, the product changes sign. There is another kind of algebra called the Jordan algebra which is commutative in the standard sense – you switch the order of two elements, the product doesn't change. Lie super-algebras are a combination of these two structures. They definitely have applications. They attracted interest and were classified when physicists became interested in so-called "supersymmetric" field theories. Actually, the Hilbert syzygy theorem involves a Lie super-algebra.

I: Is pure mathematics losing talented students to other more "glamorous" areas like mathematical finance or more applied areas like statistics and computer science?

H: This always happened to some degree. A talented person will have several areas to choose from and this has been going on a long time. Gauss had to choose between philology and mathematics. When he made some of his discoveries about the cyclotomic numbers and the construction of regular polygons, he decided that mathematics might be a better choice. Many people who could do mathematics can also do other things. Probably it's partly circumstance, what they get exposed to. I think it's also personality. For example, theoretical physicists and mathematicians tend to have very different personalities.

I: Talking about personality, there seems to be the observation that the personality of an algebraist is quite different from that of an analyst. Do you agree with that?

H: I would agree with that.

I: What do you think is the secret of your tremendous success in teaching mathematics at the university level?

H: Well, first, "tremendous success" are not words I would use. But I have worked hard to improve over the years, and it has been very rewarding to see students respond. Teaching is a complex art, and you can't sum up what you do in a few phrases. But the thing that I have worked on is to improve my communication with students. I spend a lot more time asking them questions, and less time just explaining. I sometimes say, that I used to try to show students why math is easy, and now I try to get them to see why it is hard.

I: As the Chair of the Institute's Scientific Advisory Board during the past five years, what is your deepest impression?

H: I have been extremely impressed by Louis Chen, the energy and devotion which he put into this institute. He has done a terrific job of soliciting proposals from the community in Singapore and trying to find ways in which IMS can help the mathematical community in Singapore. I think that without his leadership IMS would have been less successful.

I: I think you once mentioned the "critical mass" for active research …

H: This has been a problem and will be an on-going problem. Singapore needs expertise in mathematics and the IMS can help nurture that expertise and build it. If anybody can, Louis Chen will make that case to the Singapore government.

I: Of course, we will still need people like you to chart the direction.

H: It's really a pleasure for me to work with Louis and to help out the IMS. The whole Scientific Advisory Board has really worked extremely well together. I think we have done our best to be constructive and help make suggestions

Continued on page 18

Continued from page 17

which might make proposals stronger. It's been a pleasure working here.

I: Do you foresee continuing working for IMS for the next 5 years?

H: Well, that's up to Louis and what he wants to do. It might be good to have fresh people in to get new ideas.