Interview with Stephen Smale

George Szpiro

Editor's Note: Stephen Smale is one of the most prolific mathematicians of our time and certainly one of the most dazzling. Recipient of a Fields Medal in 1966, he was also known for his anti-Vietnam war protests and the co-founding of the Yippie movement in the late 1960s. At one time he was even subpoenaed by the House Committee on Un-American Activities. He also ran into trouble with the National Science Foundation when he declared publicly that he had done some of his best work on the beaches of Rio.

In May 2007 he was awarded, together with Hillel Furstenberg, the Wolf Prize in mathematics. In Jerusalem for the prize ceremony, Smale granted an interview in which he talked about his work and career. What follows is an edited version of the interview, conducted by George Szpiro, Israel correspondent and mathematics columnist for the *Neue Zürcher Zeitung* (Switzerland). A German version of the interview appeared in that newspaper on June 6, 2007. Szpiro's latest book is *Poincaré's Prize: The Hundred-Year Quest to Solve one of Math's Greatest Problems*, published by Dutton in June 2007.

—Andy Magid

Szpiro: *Professor Smale, why is mathematics important to you?*

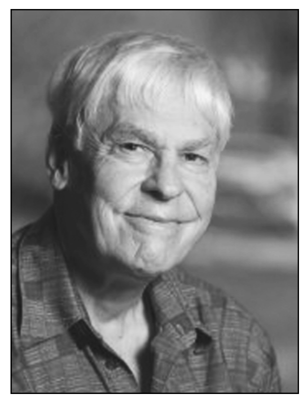
Smale: Oh, that's a tough question. Maybe I'm different from other mathematicians. I consider it as just one important thing to study. I see myself broader, as a scientist, even a little bit of an artist. So mathematics is not the sole motivating thing in my life, far from it. But I do see a beauty in mathematics because of its elegance and its ability to idealize the things you see in everyday life. Understanding the things around you has been the motivating factor for me for the past 40 years.

Szpiro: *So is mathematics a cultural endeavor?*

Smale: Well, I wouldn't say so much cultural, as science in a broad sense. The traditional motivation to do mathematics is to understand physics, but also to understand, say, economic phenomena. I am now trying to understand human vision, trying to develop something like a model for the visual cortex. Maybe it will turn out that there are some universal laws and eventually we will understand how humans learn and think. That's an example of how mathematics can help us understand natural phenomena.

Szpiro: *Why is mathematics so effective in explaining phenomena, as opposed to, say, narratives?*

Smale: Mathematics is a kind of formalized way of thinking. One can be much more precise in mathematics than in literature, express relationships in a more precise way, include magnitudes. And even fuzziness can be incorporated in mathematics by using probabilities. I use that a lot because when moving from physics to vision and biology one has to incorporate some kind of fuzziness. The way I do that is—in the mathematical tradition—by using probability.



Stephen Smale

Mathematics is so effective because one can look for universal laws more easily with mathematics than without. It enables us to abstract the main ideas. With formalization and symbols one is able to see what is universal. The abstraction allows us to see universal ideas. I have been very inspired by Newton who could see a falling apple and the motion of planets and recognize them as part of the same phenomenon. I would like to see a language that allows us to translate what we see and then recognize it as part of a broad phenomenon.

Szpiro: *Kepler's Conjecture was believed to be correct, even before it was proven, and many people believe the Riemann Conjecture to be correct. Why is it so important in mathematics to be rigorous when proving something?*

Smale: Just because a lot of people believe something does not mean it is true. I am in favor of rigorously proving big problems. On the other hand, I am not quite so devoted to the idea that proof is the most essential thing in mathematics. What may be more important are the relationships of the main structures, the concepts, and the development of these concepts. Proofs are often an important part of that but are not the main focus of my work. I'm rigorous, I try to have things correct, but sometimes proofs are almost secondary to seeing how the main structures are laid out. I look at relationships between mathematics and eventually between parts of the real world.

***Szpiro:** Do you accept computer proofs?*

Smale: Since proofs are not the ultimate in mathematics for me, computer proofs are okay. Maybe not as good as a construction, a structured conceptual proof, but okay.

***Szpiro:** Your career covered four areas: topology, dynamical systems, mathematical economics, and computer science. Why did you leave topology?*

Smale: In 1961 I did change subjects. I did not change completely, but I did leave topology. I said it publicly. I had proved Poincaré's conjecture in dimensions five and greater and I thought, after that, things were a little anti-climactic. Proofs for the third and fourth dimensions were still missing but it seemed—I won't say I was right—that these were just special cases. So it was more exciting for me to understand the dynamics of a discrete transformation of the two-sphere than working out Poincaré's conjecture.

***Szpiro:** Were you convinced at the time that Poincaré's conjecture was correct?*

Smale: Oh no, far from it. I even had a counterexample. But it did not work, I found a mistake. Whenever I work on a mathematical problem I work on both sides of the question because they reinforce each other. If you work only on one side you don't get such a good perspective. One should not have too many pre-conceived ideas. Sometimes you should say "well, if it's not true, how would you go about proving that?" Going back and forth is an important part of proving a theorem.

***Szpiro:** What did you do after topology?*

Smale: I had been doing dynamics for some years before that and had some idea about the great problems in dynamics. So I started working on those problems. Then I also did some work in electrical circuit theory, in physics, in mechanics.

***Szpiro:** How did you get into economics?*

Smale: Well I was always interested in economics because of my political activities and my contacts with so many Marxists. One day Gérard Debreu, who later received the Nobel Prize for economics, came to me and asked me some mathematical questions about equilibria, and I told him about Sard's theorem, which was relevant to his research. A friendship grew between us. I learned a lot from him and he from me. We never worked together but we talked a lot. In fact, I helped him get the Nobel Prize. Ken Arrow and I nominated him to the Nobel committee.

***Szpiro:** Then you left economics and got into the field of algorithms.*

Smale: Yes, after a few years. I had developed algorithms to find economic equilibria. I was not trying to simulate. I was just trying to find an abstract mathematical algorithm; other people simulated it. Given supply and demand, the task was to find the equilibrium prices in the economy. And I was doing it in the general setting of a possible economy. There was another algorithm by Herbert Scarf. I believe mine was faster and more natural. This led to the question, Which was better? So I continued towards computer science in order to understand why one algorithm is better than another.

***Szpiro:** Was your algorithm supposed to describe the workings of the economy?*

Smale: No, not really. There are a couple of issues here. One of them is, how the economy works, how prices adjust. For me that was the biggest unsolved problem in economics. I spent time on that and failed. There is another problem. If parameters change, how do economic agents find the changing equilibrium? How do they locate it numerically? And I did the theory, the algorithm, of how to do that.

***Szpiro:** Was it your aim to aid a centralized economy find the equilibrium?*

Smale: I was never quite so strong on that. As a student, before I was a Vietnam war protester, I was also a communist, but never because of the economies of Vietnam or Russia. I did not know that much about economics, and I was already somewhat disenchanted by that. As I got older I abandoned Marxism. But it took many years, experience of the world, maturing intellectually, and seeing what was happening. I was never interested in Marxism from the point of view of a planned economy.

Later, I became interested in understanding markets. But I am not a believer in the capitalist system, far from it. Let's say that over the years I became market-oriented. So when I got into algorithms, I was inspired by the market economy; on the assumption that the market gives us equilibria, how does one find them? They are given by equations, and I was providing algorithms for solving these equations.

***Szpiro:** What are you working on now?*

Smale: While in Israel [for the Wolf Prize ceremony] I will give three talks. At the Weizman Institute I will talk on the mathematics of vision. It is some kind of visual cortex model, but more universal. Then I go to Haifa University where I will speak on data, the geometry of data. One sees all these data points and wants to find an underlying geometry. So I am going back to topology a little to do that. Data is the main thing one is trying to understand, and I am looking at the geometry, or the topology, of data. It is not quite pattern recognition; my first talk is connected to pattern recognition. All these things are a little bit mixed together but they are different.

Then I will talk in Beersheva on the flocking of birds. It's a big thing in zoology, there are lots of observational studies. You have a bunch of birds on the ground, and then they suddenly all go up in the air and fly together at the same speed. It has to do with control theory, and robotics people want tiny robots to communicate with each other. They are the same phenomena. So they want to see how they can organize these kinds of common phenomena. A similar phenomenon occurs when a language emerges. The idea is how one can reach common understanding through the senses. In economics it would be the belief in a common price system, a necessary condition for prices to operate. So it goes back to my old question in economics: How do people arrive at a common belief in a price system?

Szpiro: *You have observed mathematics for half a century. Where do you think the field is going?*

Smale: My feeling is that there is a shift in mathematics away from traditional areas of physics. It used to be a big area for mathematics and for thousands of years inspired a lot of mathematics. But mathematicians seem focused too much on physics. I believe that things are changing much more in mathematics than in physics. Like the areas that I work in, like vision and the other questions coming in from biology, statistics, engineering, computer science, and especially computation. A lot of these things influence the way that mathematics is changing. So where is mathematics going? It is leaving physics to a great extent and moving into the areas I just mentioned.

Szpiro: *These are areas of applied mathematics. What about pure mathematics?*

Smale: I am not talking about applied mathematics. I don't believe in that dichotomy. I am talking about using mathematics to understand the world. When developing calculus and differential equations Newton was doing mathematics in order to understand the laws of gravitation. Did he do applied math? I don't think so. Did he do pure math? No. So that's the kind of mathematics I am thinking of. It's not what it was 150 years ago. Problems come down more from computer science, engineering, and biology. But it's mathematics proper, it's not applications.