

1. Exploring Nullspaces

- (a) The **column space** of a matrix is the **range** or possible outputs of a transformation/linear operation/function. It is also the **span** of the vectors that form the columns of the matrix.
- (b) The **nullspace** is the set of input vectors that output a zero vector

For the following five matrices, answer the following questions:

- (a) What is the column span of A? What is its dimension?
- (b) What is the nullspace of A? What is its dimension?
- (c) (optional) Do the columns of A form a basis of \mathbb{R}^2 ? Why or why not?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} -2 & 4 \\ 3 & -6 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 3 \\ 0 & -1 & -2 \end{bmatrix}$