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- A. 1472 B. 1508 C. 1512 D. 1548 E. 1705

11. Three people (X, Y, Z) are in a room with you. One is a knight (knights always tell the truth), one is a knave (knaves always lie), and the other is a spy (spies may either lie or tell the truth). X says, "I am the knight." Y says, "X isn't lying." Z says, "I am the spy." Which of the following correctly identifies all three people?

- | A. | B. | C. | D. | E. |
|------------------|------------------|------------------|-----------------|------------------|
| X is the knave. | X is the spy. | X is the knight. | X is the knight | X is the knave. |
| Y is the knight. | Y is the knave. | Y is the knave. | Y is the spy. | Y is the spy. |
| Z is the spy. | Z is the knight. | Z is the spy. | Z is the knave. | Z is the knight. |

12. If $(f, 0)$ is the focus of the parabola given by $y^2 = 12x$, and (h, k) is the center of the hyperbola given by $x^2 - y^2 - 6x - 8y = 8$, find $f + h + k$.

- A. -2 B. -1 C. 0 D. 1 E. 2

13. For how many $a \in (0, 1)$ will $\sin(x) + a \sec(x) = \cos(x) + a \csc(x)$ have at least two distinct solutions on $(0, 2\pi)$?

- A. 0 B. 2 C. 4 D. 6 E. Infinitely many

14. The equation $a^4 + 2b^2 + c^2 = 2022$ has six solutions where a, b, c are positive integers. The values of a and c are both even for all six solutions. Find the only value of b that appears in two different solutions.

- A. 1 B. 5 C. 11 D. 19 E. 29

15. A rectangular tabletop is made from 1000 square tiles of the same size with no space between them. It has dimensions 28" by 17.5". What percentage (rounded to the nearest tenth of a percent) of the total area of the tabletop consists of edge tiles (that is, tiles that have one or two edges along the outside edge)?

- A. 11.5% B. 12.2% C. 12% D. 12.6% E. 13%

16. (i) Is the binary operation $a * b = a + b - ab$ commutative for all real numbers a, b ?

(ii) Is the binary operation $a * b = a + b - ab$ associative for all real numbers a, b ?

- A. (i)Yes (ii)Yes B. (i)Yes (ii)No C. (i)No (ii)Yes D. (i)No (ii)No E. Impossible to determine

17. The position of the hands of a clock are recorded at a moment in time between 4:25:00 and 5:00:00. Call this time HR:MIN:SEC. 33 minutes and 25 seconds later, the angle measured clockwise from the hour hand to the minute hand is 10 times greater than it was when the positions were first recorded. To the nearest second, what is MIN + SEC?

- A. 57 B. 61 C. 65 D. 67 E. 77

18. A basketball team plays eight games in a tournament against evenly matched teams (so we may assume that the probability of the team winning each of the eight games is $\frac{1}{2}$). Find the probability that the team wins at least three games in a row or loses at least three games in a row at least once during the tournament.

- A. 31/64 B. 35/64 C. 39/64 D. 43/64 E. 47/64

19. Nine players are playing Texas Hold 'Em using a standard deck of 52 cards (2-10, J, Q, K, A in each of 4 suits). Each player is dealt two cards, then five cards are dealt face up for all players to share. Suppose Joe is dealt two hearts. What is the probability, to the nearest %, that he will make a heart flush (which means that there will be at least three more hearts dealt face up)?

- A. 3% B. 4% C. 5% D. 6% E. 7%

20. Al and Bob are at opposite ends of a diameter of a silo in the shape of a tall right circular cylinder with radius 150 ft. Al is due west of Bob. Al begins walking along the edge of the silo at 6 ft per second at the same moment that Bob begins to walk due east at the same speed. The value closest to the time in seconds when Al first can see Bob is:

- A. 46 B. 47 C. 48 D. 49 E. 50