Test #2 **AMATYC Student Mathematics League** **Spring 2025**

1. Define a binary operation # on the integers by $m \# n = m^2 - n$. Find 2#(0#(2#5)).
A. -1 B. 0 C. 1 D. 3 E. 5

2. Let *S* be the set of all possible strings of length five comprised of only the digits 0,1,2,3,4 (including those with one or more 0s as the first digit(s)). For example, 44444, 00023, 12220, and 01234 are all elements of *S*. How many elements of *S* are *even* when considered as a Base 5 number, but *not even* when considered in Base 10?
A. 624 B. 625 C. 955 D. 1228 E. 1229

3. Five fair dice are rolled at the same time (so order does not matter). What is the probability that the five results form a "large straight" (either 12345 or 23456)? Round your answer to the nearest ten thousandth.
A. 0.0003 B. 0.0039 C. 0.0309 D. 0.0926 E. 0.3926

4. The graph of $x^2 + 2xy + y^2 = 0$ is…
A. an ellipse. B. a parabola. C. a hyperbola. D. a single line. E. two intersecting lines.

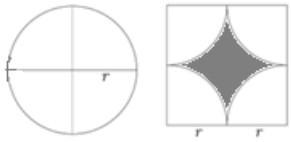
5. There are positive integers *m* and *n* such that $\sum_{i=1}^{9} i^m = 2025$ and $(\sum_{i=1}^{9} i)^n = 2025$. Find $m + n$.
A. 2 B. 3 C. 4 D. 5 E. 6

6. How many zeros does the number $\sum_{i=1}^{9999} i^3$ end in?
A. 3 B. 4 C. 5 D. 6 E. 8

7. Suppose that the line $x + y = 2$ is the perpendicular bisector of the line segment with endpoints (0, 0) and (a, b), and the line $y - x = 4$ is the perpendicular bisector of the line segment with endpoints (c, d) and (a, b). Find the equation of the line that is the perpendicular bisector of the line segment with endpoints (c, d) and (0, 0).
A. $3y - x = 10$ B. $x + 3y = 8$ C. $3y - x = 8$ D. $x + 3y = 10$ E. $x - 2y = 6$

8. Four real numbers, $a < b < c < d$ can be paired in six different ways. If each pair has a different sum, and if the four least sums are 1, 2, 3, and 4, what is the least possible value for *d*?
A. 2 B. 5/2 C. 7/2 D. 4 E. 9/2

9. What is the ratio of the area of a circle to the shaded area of the figure created by reflecting each quarter circle around the chord connecting its endpoints?
A. $\frac{3\pi}{4}$ B. $\pi - 1$ C. $\frac{\pi}{\pi-3}$ D. $\pi + 1$ E. $\frac{\pi}{4-\pi}$



10. Three people (X, Y, Z) are in a room with you. One is a knight (knights always tell the truth), one is a knave (knaves always lie), and the other is a spy (spies may either lie or tell the truth). X says, "I am not a spy." Then Y says, "I am a spy." And finally, Z says, "Y is lying." Which of the following correctly identifies all three people?

A.	B.	C.	D.	E.
X is the knave Y is the spy Z is the knight	X is the spy Y is the knave Z is the knight	X is the knight Y is the spy Z is the knave	X is the knight Y is the knave Z is the spy	Impossible to determine from given information.

11. The ellipse $x^2 + 4y^2 - 16y = -15$ is translated (shifted) downward until it first touches the graph of $y = x^2$. What is the y-coordinate of the center of the translated ellipse?
A. 7/8 B. 15/16 C. 1 D. 17/16 E. 9/8

12. What is the greatest integer n such that 2025^n divides 2025!?

A. 55 B. 101 C. 225 D. 252 E. 505

13. An ordered partition of n is an ordered sum of positive integers which equals n. For example, $2 + 2 + 1 + 2$, $4 + 3$, and $3 + 4$ are all *distinct* ordered partitions of 7. How many distinct ordered partitions of 10 are there which have no term greater than 4?

A. 372 B. 377 C. 401 D. 448 E. 512

14. A quadrilateral is formed using the x-intercepts and y-intercepts of the circle $x^2 - 38x + y^2 - 66y = 575$ as vertices. What is the area of the quadrilateral?

A. 1664 B. 2496 C. 2964 D. 3264 E. 3664

15. Suppose that $f(x) = \frac{x^2-16}{ax+b}$ for some real numbers a and b, and that $f(x)$ has an oblique asymptote (a.k.a. slant asymptote) of $y = 3x + 7$. Find $f(-3)$.

A. 65/9 B. 63/16 C. 35/9 D. –7/16 E. –35/9

16. In a certain game, a player rolls a fair six-sided die until they roll a 6, and then adds all of the results (including the final 6). For example, if the player rolls 2, 3, then 6, the sum is 11. What is the expected value of this sum?

A. 12 B. 16 C. 18 D. 20 E. 21

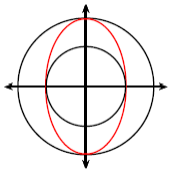
17. How many 8-digit whole numbers with no digits repeated are multiples of 9? Note that a number cannot start with 0 here.

A. 12,624 B. 68,932 C. 104,682 D. 162,668 E. 181,440

18. Define the sequence $\{a_n\}$ as follows: $a_0 = 5$, $a_1 = 2$, and for $n \geq 2$, $a_n = a_{n-1} - a_{n-2}$. Find $\sum_{n=0}^{2025} a_n$.

A. –3 B. –1 C. 4. D. 5 E. 7

19. A circle of radius r_1 is inscribed in an ellipse, and that ellipse is inscribed in another circle of radius r_2. If the equation for the ellipse is $x^2 + by^2 = ab$, for $a, b > 0$, what is the product $r_1 r_2$?



A. $a\sqrt{b}$ B. $b\sqrt{a}$ C. $\sqrt{ab}$ D. $\frac{\sqrt{b}}{a}$ E. $\frac{\sqrt{a}}{b}$

20. In the quadrilateral PQRS, PQ = 1, QR = RS = $\sqrt{2}$, PS = $\sqrt{3}$, and QS = 2. If T is the point of intersection of the diagonals QS and PR, find the measure in degrees of angle RTS.

A. 45 B. 55 C. 60 D. 75 E. 105