Section 9.4

In general, if $P(t)$ is the value of a quantity y at time t and if the rate of change of P with respect to t is proportional to its size $P(t)$ at any time, then

$$\frac{dP}{dt} = kP.$$

This equation is called the **law of natural growth**.

The solution of the initial value problem

$$\frac{dP}{dt} = kP \qquad P(0) = P_0$$

is

$$P(t) = P_0 e^{kt}.$$

Definition: If the relative growth rate decreases as the population P increases and becomes negative if P ever exceeds its **carrying capacity** M, the maximum population that the environment is capable of sustaining in the long run.

The model for population growth is known as the **logistic differential equation**:

$$\frac{dP}{dt} = kP\ (1 - \frac{P}{M}).$$

Thus, the solution to the logistic equation is

$$P(t) = \frac{M}{1+Ae^{-kt}}, \text{ where } A = \frac{M-P_o}{P_o}.$$

Exercise 1. Write the solution if the initial-value problem

$$\frac{dP}{dt} = 0.08P(1 - \frac{P}{1000}),\ P(0) = 100$$

and use it to find the population sizes $P(40)$ and $P(80)$. At what time does the population reach 900? (Section 9.4 Stewart Example 2)

Exercise 2. Find an equation for the population of Reedley, if the relative growth rate is 0.05, and the current population is 20,000. (DR 6.5 #1)

Exercise 3. Find a solution to the following logistic differential equation:

$$\frac{dP}{dt} = 0.15P\ (1 - \frac{P}{120}).$$

(DR 6.5 #3)

Class Exercise 1. A 2000-gallon tank can support no more than 150 guppies. Six guppies are introduced into the tank. Assume that the rate of growth of the population is

$$\frac{dP}{dt} = 0.0015P(150 - P),$$

where t is in weeks.

(a) Find a formula for the guppy population in terms of t.
(b) How long will it take for the guppy population to be 100? 125? (Waits Section 6.5 #17)

Class Exercise 2. Suppose a population $P(t)$ satisfies

$$\frac{dP}{dt} = 0.4P - 0.001P^2 \qquad P(0) = 50$$

where t is measured in years.
(a) What is the carrying capacity?
(b) What is $P'(0)$?
(c) When will the population reach 50% of the carrying capacity? (Section 9.4 #4)

Class Exercise 3. (a) Assume that the carrying capacity for the US population is 4 billion. Use it and the fact that the population was 250 million in 1990 to formulate a logistic model for the US population.
(b) Determine the value of k in your model by using the fact that the population in 2000 was 275 million.
(c) Use your model to predict the US population in the years 2100 and 2200.
(d) Use your model to predict the year in which the US population will exceed 350 million. (Section 9.4 #8)

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