Exam #3 April 1993

1. If the line in the xy-plane through the points (3, p) and (p, -2) has slope $\frac{-3}{2}$, then

   A. $p \le -10$ B. $-10 < p \le 0$ C. $0 < p \le 10$ D. $10 < p \le 20$ E. $p > 20$

2. Driving along a level, straight road, I see a tower directly ahead whose top has an angle of elevation of 45° with the ground. After driving 200 ft, I find the tower's top has an angle of elevation of 60°. How tall (to the nearest 10 ft) is the tower?

   A. 270 ft B. 280 ft C. 400 ft D. 470 ft E. 480 ft

3. The region in the xy-plane determined by the inequalities $x \ge 0$, $y \ge 0$, $x \le 4$, $y \le 4$, $3x - 4y + 8 \ge 0$ is a

   A. triangle B. square C. nonsquare rectangle D. trapezoid E. pentagon

4. If AM + AT + YC = ACT, each number is in standard decimal form, and each letter represents a unique decimal digit, then which value CANNOT equal T?

   A. 0 B. 3 C. 5 D. 7 E. 8

5. If $f(x) = -x^2 + 2|x^2 + x|$, then $f(-3) =$

   A. 0 B. 3 C. 15 D. 21 E. 33

6. A farmhand agrees to work for one year, after which he is to receive $1700 and a tractor. He quits after 9 months, and must pay $100 to receive the tractor. If he worked one more month, he should receive the tractor and

   A. $500 B. $600 C. $700 D. $800 E. $1000

7. For how many positive real numbers u does there exist at least one real number v for which (log u)(log v) = log (uv) ?

   A. none B. one C. a finite number > 1 D. all but one E. all

8. In a group of 20 people, 17 own a TV, 15 own a car, and 14 own a bike. What is the largest number of people who could own a stereo, if no one owns all four items?

   A. 6 B. 8 C. 10 D. 12 E. more than 12

9. The last two digits of 19^{93} are A. 61 B. 59 C. 39 D. 21 E. 19

10. The fundamental period of the function $f(t) = 4 \sin 2t$ is

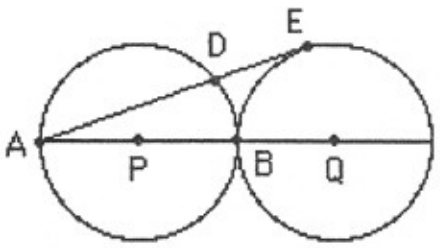
    A. $\frac{\pi}{4}$ B. $\frac{\pi}{2}$ C. π D. 2π E. 4π

11. Three fair coins are labelled 1, 3, and 5 respectively on one side and 2, 4, and 6 respectively on the other. In flipping these coins, the probability that the total of the three numbers which are showing is 11 equals

    A. 0 B. $\frac{1}{4}$ C. $\frac{3}{8}$ D. $\frac{1}{2}$ E. $\frac{5}{8}$

12. Al says, "I didn't do it," Bo says, "Cy did it," Cy says, "Ed did it," and Ed says, "Al or Cy did it." If at least three of the men are telling the truth, who did it?

    A. Al B. Bo C. Cy D. Ed E. none of them

13. 

Circles P and Q both have radius 6 and are mutually tangent at B. If $\overline{AE}$ is tangent to circle Q at E and intersects circle P at D, then DE =

A. $4\sqrt{2}$ B. $3\sqrt{3}$ C. 4 D. 5 E. 6

14. A quadratic polynomial P(x) satisfies $P(x) = \left[\frac{P(x+1) - P(x-1)}{2}\right]^2$ for all real x. The value of [P(0) - P(-1)] + [P(0) - P(1)] is

A. -1 B. $\frac{-1}{2}$ C. 0 D. $\frac{1}{2}$ E. 1

15. If $f(x) = x^2 - 1$, how many distinct real solutions does the equation f(f(f(x))) = 0 have?

A. 2 B. 3 C. 4 D. 6 E. 8

16. Find the sum of all possible five-digit numbers in which each of the digits 1, 2, 3, 4, and 5 appears exactly once. Write this sum in the corresponding blank on the answer sheet.

17. For a certain number t, the values tan t, cos t, and sec t form the third, fourth, and fifth terms respectively of an arithmetic sequence. For what value k is cot t the kth term of this sequence?

A. 1 B. 2 C. 6 D. 7 E. 10

18. Let $f(x) = \frac{1}{x^2}$. If the area of the region in the first quadrant of the xy-plane bounded by the y-axis, the line $y = \frac{1}{4}$, the line y = 1, and y = f(x) is 1, and the area of the region bounded by the x-axis, the line x = 1, the line x = 2, and y = f(x) is $\frac{1}{2}$, then the area bounded by y = f(x) and the lines y = x and 8y = x is

A. $\frac{1}{4}$ B. $\frac{3}{8}$ C. $\frac{1}{2}$ D. $\frac{3}{4}$ E. 1

19. The four vertices of a quadrilateral lie on a circle of diameter 25. If three consecutive sides have lengths 20, 24, and 15 respectively, then the fourth side has length

A. 5 B. 7 C. 16 D. 21 E. none of these

20. Assume grass grows at a constant rate and sheep eat grass at a constant rate. If eight sheep can completely eat a pasture in five days, and nine sheep can completely eat the same pasture in four days, what is the maximum number of sheep the pasture can sustain indefinitely?

A. 1 B. 2 C. 3 D. 4 E. 5