Feb 4

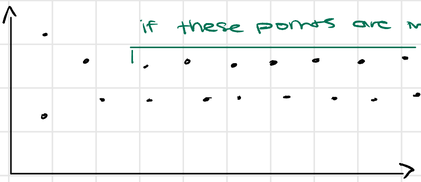
Subsequences

Definition: Let S_n $_{n \in N}$ be a sequence
Let n_k be a strictly increasing seq in $\mathbb{N}$
$n_1 < n_2 < n_3 \ldots$
then, we define the subsequence as
$t_k := S_{n_k}$ for $k = 1, 2, \ldots$

can be an infinite subsequence

THM: Let S_n be any sequence, and $t \in \mathbb{R}$
then, S_n has a subsequence converge to t
<u>if and only if</u> $\forall \varepsilon > 0$, $A_\varepsilon = \{ n \in \mathbb{N} \mid |S_n - t| < \varepsilon \}$
is infinite

Ex:



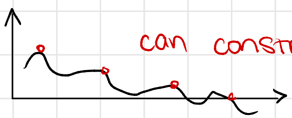
→ we can construct our subsequence to contain only these points and thus converge

Feb 9

THM: Every seq (S_n) has a monotone subsequence

Case 1: there are infinitely many dominant terms

S_n is dominant if $\forall m > n$ $S_n > S_m$



Case 2: there are finitely many dominant terms
↳ we can construct a monotonic increasing sequence

THM: Every bounded seq has a convergent subseq
Find monotonic subseq → bounded → convergent

Def: Subsequence Limit
t is a subsequence limit if s_n has a subseq that converges to t

$\limsup s_n$ and $\liminf s_n$ are subseq limits