January 19: Lecture 1

$\mathbb{N}$: the Naturals	$\{1, 2 \ldots\}$ operations: $+$, $\times$ has an ordering
$\mathbb{Z}$: the "zintegers"	$\{\ldots -3, -2, -1, \underline{0}, \underline{1}, 2, 3 \ldots\}$ 0 special since $0+a=a$; 1 special since $1 \cdot a = a$ operations: $+$, $\times$, $-$ example of a "ring"
$\mathbb{Q}$: the rationals	$\{\frac{n}{m} \mid n, m \in \mathbb{Z}, m \neq 0\}$ operations: $+$, $\times$, $-$, $\div$ example of an "ordered field"
$\mathbb{R}$: the reals	$\mathbb{R} \supset \mathbb{Q} \supset \mathbb{Z} \supset \mathbb{N}$ satisfies <u>completeness Axiom</u> Every nonempty subset A of $\mathbb{R}$ that is bounded above has a least upper bound sup(A) exists and is a real number
Induction	show claim holds for $n=1$ show holds for $n+1$ EX: $7 \mid 11^n - 4^n$ $n=1$ $\quad 11-4=7 \quad 7 \mid 7$ clearly divisible $\quad$ case for n is true $11^{n+1} - 4^{n+1} = 11 \cdot 11^n + 4 \cdot 4^n = 7 \cdot 11^n + 4(11^n - 4^n)$

Rational Numbers	$r = \frac{c}{d} \in \mathbb{Q}$ is a rational number and r satisfies $c_n x^n + c_{n-1} x^{n-1} + \dots + c_0 = 0$ $c_i \in \mathbb{Z} \quad c_n \neq 0 \quad c_0 \neq 0$ then, $d \mid c_n$ $c \mid c_0$ To determine if # is rational, check