January 21 Foundations

Max

$\underline{\alpha \in S}$ is a maximum if $\forall \beta \in S$, $\alpha \geq \beta$

Min

$\underline{\alpha \in S}$ is a minimum if $\forall \beta \in S$, $\alpha \leq \beta$

not guaranteed to exist

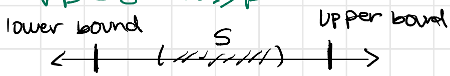
Key example:

$S = [0, \sqrt{2})$ $S \subset \mathbb{Q}$

$\sqrt{2} \notin \mathbb{Q}$ $\rightarrow$ no max

Upper Bound

$\alpha \in \mathbb{R}$ is an upper bound of S if $\forall \beta \in S$ $\alpha \geq \beta$



Lower Bound

$\alpha \in \mathbb{R}$ is a lower bound of S if $\forall \beta \in S$ $\alpha \leq \beta$

Sup

$S \neq \emptyset$ $S \subset \mathbb{R}$

If S is bounded above, the sup is the least upper bound

If max exists, sup = max

Inf

the least lower bound

if min exists, inf = min

Completeness Axiom

Every nonempty subset A of $\mathbb{R}$ that is bounded above has a least upper bound

$\sup(A)$ exists and is a real number

↳ extension to inf: consider $-S = \{-x \mid x \in S\}$

$\inf(S) = -\sup(-S)$

Archimedian Property

If $a > 0$, $b > 0$ are real, then for some $n \in \mathbb{N}$,

$n \cdot a > b$, $a > \frac{b}{n}$

What does this mean?

consider some really small $a \in \mathbb{R}$

no matter how small $\exists n \in \mathbb{N}$ s.t. $a > \frac{1}{n}$

Sequence and Limits

Sequence

a collection of numbers indexed by $n \in \mathbb{N}$ a function $\mathbb{N} \to \mathbb{R}$

$a_1, a_2, a_3 \dots$ $a_n \in \mathbb{R} \ \forall n \in \mathbb{N}$

TO BE CONTINUED