Chapter 7 pg 151-154

Uniform Convergence & Integration

7.16 Thm) Let α be monotonically increasing on $[a,b]$. Suppose $f_n \in \mathcal{R}(\alpha)$ on $[a,b]$, for $n=1,2,3,\ldots$, and suppose $f_n \to f$ uniformly on $[a,b]$. Then $f \in \mathcal{R}(\alpha)$ on $[a,b]$ and

$$\int_a^b f\,d\alpha = \lim_{n\to\infty} \int_a^b f_n\,d\alpha$$

Corollary If $f_n \in \mathcal{R}(\alpha)$ on $[a,b]$ then

$$\int_a^b f\,d\alpha = \sum_{n=1}^{\infty} \int_a^b f_n\,d\alpha$$

$\Rightarrow$ series may be integrated term by term.

Uniform Convergence & Differentiation

7.17 Thm) Suppose $\{f_n\}$ is a sequence of functions differentiable on $[a,b]$ and s.t. $\{f_n(x_0)\}$ converges for some point x_0 on $[a,b]$

If $\{f_n'\}$ converges uniformly on $[a,b]$, then $\{f_n\}$ converges uniformly on $[a,b]$ to a function f and

$$f'(x) = \lim_{n\to\infty} f_n'(x) \quad (a \le x \le b)$$

7.18 Thm) There exists a real continuous function on the real line which is <u>nowhere</u> differentiable.