ch 1 sec 1

Set of Natural Numbers $\mathbb{N}$

1) 1 belongs to $\mathbb{N}$
2) If n belongs to $\mathbb{N}$ => it's successor $n+1$ belongs to $\mathbb{N}$
3) 1 is not the successor of any elem in $\mathbb{N}$
4) If n & m have same successor => $n=m$
5) A subset of $\mathbb{N}$ which contains 1, & which contains $n+1$ whenever it contains n, must equal $\mathbb{N}$

Sec 2 The set $\mathbb{Q}$ of Rational Numbers

A number is called an algebraic number if it satisfies a polynomial equation. $c_n x^n + \cdots + c_1 x + c_0 = 0$

Rational Zeros theorem

Only rational candidates for solutions of poly eqn have the form c/d where c divides c_0 & d divides c_n.

Sec 4: The completeness Axiom

Let S be a non-empty subset of $\mathbb{R}$:

- s_0 is the max of S [$s_0 = \max S$] iff $s_0 \in S$ and $s_i \leq s_0$ $\forall s_i \in S$
- If S contains a smallest elem => min of S → $\min S$.

Let S be a non-empty subset of $\mathbb{R}$

- If a real # M satisfies $s \leq M$ $\forall s \in S$, => M is called upper bound of S & set S is bounded above by M
- If $m \leq s$ $\forall s \in S$ => lowerbound
- S is "bounded" if has lower & upper bound.

Completeness Axiom

Every nonempty subset S of $\mathbb{R}$ that is bounded above has a supremum so $\sup S$ exists & is Real.

Section 7 - Limit of Sequence

Sequence is a function whose domain is a set in form
$\{n \in \mathbb{Z} : n \geq m\}$ denoted as s_n.

- A sequence (s_n) of real #'s is said to converge to the real # s provided that

for each $\varepsilon > 0$ $\exists$ N s.t. $n > N \Rightarrow |s_n - s| < \varepsilon$

Section 9

Theorem: Convergent Sequences are Bounded

Theorem: If the seq (s_n) converges to s & k is in $\mathbb{R}$
$\Rightarrow$ (ks_n) converges to ks.
$\lim ks_n = k \cdot \lim s_n$

- $\lim(s_n + t_n) = \lim(s_n) + \lim(t_n)$
- $\lim(s_n t_n) = \lim(s_n) \lim(t_n)$

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