Math 104 Midterm 1 Feb 17, 2022

Name: ______________________________

In the following lim means $\lim_{n\to\infty}$, and $\sum_n$ means $\sum_{n=1}^{\infty}$.

1. (10 points each) True or False. If you think the following statement is true, give a proof; if you think it is false, give a counter-example.
   (a) Let $A = \sqrt{2}\mathbb{Q} = \{\sqrt{2}x \mid x \in \mathbb{Q}\}$, then for any non-empty open interval $(a, b) \subset \mathbb{R}$, we have $\sup(A \cap (a, b)) = b$.
   (b) Let $(s_n)$ be a sequence in $\mathbb{R}$, such that $\lim s_n = s \in \mathbb{R}$. Let $t_n = s_n - s_{2n}$. Then, $\lim t_n = 0$.
   (c) Let $(s_n)$ be any bounded sequence in $\mathbb{R}$, and let $S$ be the set of subsequential limits of $s_n$. Then $S$ is countable.
   (d) Let $(s_n), (t_n)$ be two bounded sequences in $\mathbb{R}$, then $\limsup(s_n + t_n) = \limsup s_n + \limsup t_n$
   (e) Let $(a_n), (b_n)$ be two bounded non-negative sequences, and $\sum_n b_n$ is convergent. Then $\sum_n a_n b_n$ is convergent.
   (f) If the sequence$(s_n)$ converges to 1, and $t$ is a subsequential limit of $(t_n)$, then $t$ is a subsequential limit of $(t_n s_n)$.

2. (10 points each) Determine if the following limit or series are convergent or not, give your reasoning (you don't have to compute the actual value) (10 points each).
   (a) $\sum_{n=0}^{\infty}(-1/3)^n \sin(n)$
   (b) $\lim(\sqrt{n})^{1/n}$.
   (c) $\sum_n 1/\sqrt{n!}$
   (d) $\sum_n (1/n)^{1+1/n}$