Math 104 HW

Ross

34.2

(a) $\lim_{x\to 0} \frac{1}{x}\int_0^x e^{t^2}\,dt$ is the average height of the function $f(t)=e^{t^2}$ from 0 to x.

So as $x\to 0$ this averages to $f(0)=e^0=\boxed{1}$

(b) $\lim_{h\to 0} \frac{1}{h}\int_3^{3+h} e^{t^2}\,dt$ is the average height of $f(t)=e^{t^2}$ from 3 to $3+h$

So as $h\to 0$, this averages to $f(3)$

$=\boxed{e^9}$

34.5

$F'(x)=f(x+1)-f(x-1)$

by Fundamental thm of Calculus.

$\int_{x-1}^{x+1} F(t)\,dt = \int_0^{x+1} F(t)\,dt - \int_0^{x-1} F(t)\,dt$ both of which are differentiable on $\mathbb{R}$

$\Rightarrow$ difference is differentiable on $\mathbb{R}$.

34.7

$u=\sqrt{1-x^2} \Rightarrow du=\frac{1}{2}(1-x^2)^{-1/2}\cdot(-2x)\,dx = -x(1-x^2)^{-1/2}\,dx$

$\Rightarrow dx=-\frac{1}{x}(1-x^2)^{1/2}\,du = -\frac{u}{x}\,dx$

$\int_0^1 x\sqrt{1-x^2}\,dx = \int_1^0 -u\,du = -\frac{u^2}{2}\Big|_1^0 = \boxed{\frac{1}{2}}$

34.7

$u=1-x^2 \Rightarrow du=-2x\,dx \Rightarrow dx=-\frac{1}{2x}\,du$

$\Rightarrow \int_0^1 x\sqrt{1-x^2}\,dx = -\frac{1}{2}\int_1^0 \sqrt{u}\,du = -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_1^0 = -\frac{1}{2}\cdot\left(-\frac{2}{3}\right)=\boxed{\frac{1}{3}}$