104 HW 5

13.3

(a) $d(x,y) = \sup\{|x_j - y_j| : j = 1,2,\dots\} = \sup\{|y_j - x_j| : j = 1,2,\dots\} = d(y,x)$

since $|x_j - y_j| = |y_j - x_j|$

$d(x,y) = \sup\{|x_j - y_j| : j = 1,2,\dots\}$

so $|x_j - y_j| \geq 0$ is a lower bound for sup $\Rightarrow d(x,y) \geq 0$

$d(x,z) = \sup\{|x_j - z_j| : j = 1,2,\dots\} = \sup\{|x_j - y_j + y_j - z_j| : j = 1,2,\dots\}$

$\leq \sup\{|x_j - y_j| + |y_j - z_j| : j = 1,2,\dots\} \leq \sup\{|x_j - y_j| : j = 1,2,\dots\}$
$+ \sup\{|y_j - z_j| : j = 1,2,\dots\}$

$= d(x,y) + d(x,z)$

So d is a metric for B

(b) No since consider the sequences $(0,0,0,\dots) = x$
$(1,1,1,\dots) = y$

$d(x,y) = \infty \notin \mathbb{R}$ so the distance metric isnt even defined properly.

13.5

(a) consider an element $u \in \bigcap\{S \setminus U : U \in \mathcal{U}\}$

$\Rightarrow u \in S \setminus U \quad \forall\, U \in \mathcal{U}$

$\Rightarrow u \notin U \quad \forall\, U \in \mathcal{U}$

$\Rightarrow u \notin \bigcup\{U : U \in \mathcal{U}\}$

$\Rightarrow u \in S \setminus \bigcup\{U : U \in \mathcal{U}\}$

Consider an element $u \in S \setminus \bigcup\{U : U \in \mathcal{U}\}$

$\Rightarrow u \notin \bigcup\{U : U \in \mathcal{U}\}$

$\Rightarrow u \notin U \quad \forall\, U \in \mathcal{U}$

$u \in S \setminus U \quad \forall\, U \in \mathcal{U}$

$\Rightarrow u \in \bigcap\{S \setminus U : U \in \mathcal{U}\}$

(b) In demorgan's law, consider U's to be open sets

$\Rightarrow S \setminus U$ is a closed set.

the left hand side is an intersection of many closed sets

Since the Union of finitely many open sets is an open set, The right hand side becomes $S \setminus$ open set = closed set.

So the intersection of any collection of closed sets is a closed set.

13.7

~~Lemma~~

Let U be the open set

for $x \in U$ consider ~~the open set~~

the smallest $y_1 \geq x$ s.t. $y_1 \notin U$

Consider the largest $y_2 \leq x$ s.t. $y_2 \notin U$

Then the open ball with center x, radius $\min(|x-y_1|, |x-y_2|)$ is fully contained in U.

By definition x has an open ball entirely in U. So consider all these open balls (unioning 2 if disjoint)

Consider all $(y_1, y_2) \in U$. The union of these forms U.

(4) It is clear that $S_1 \subseteq S_2$ since we can consider the constant sequence for all elements in S_1.

Now we just need to show an element of S_2 is in S_1

Consider $x \in S_2$. By definition there is some sequence P_n in S_1 that converges to x

$P_1 \quad P_2 \quad P_3 \ldots$

Now write out the sequences from S that formed $P_1, P_2, \ldots$ as limits

$x_{11} \; x_{21} \ldots$
$x_{21} \; x_{22} \ldots$
$\vdots$

we can take the diagonal sequence to get a new one converging to x $\Rightarrow x \in S_1$

So $S_1 = S_2$

(5) Since $\bar{S}$ is closed and it contains S, it remains to show that all closed subsets in X that contain S also contain $\bar{S}$.

If a closed subset of X contains S, by definition it must contain all of S's subsequential points $\Rightarrow$ it contains $\bar{S}$ so we are done.

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