104 HW

(1) We know X ~~[illegible]~~ is open cover compact so every open cover of it admits a finite subcover.

~~[illegible]~~ by just appending a fixed y, for all $y \in Y$ we have a separate finite cover of ~~[illegible]~~ $(x, y)\ \forall x \in X$

Since every open cover of Y also has a finite subcover, we can consider the union of all these to get an open cover of Y considering only the right element. However, we do have a finite subcover of this covering since Y is compact. So, this new subcover ~~[illegible]~~ covers $X \times Y$ and it is finite

$\Rightarrow X \times Y$ is compact.

(2) (a) $\boxed{\text{False}}$ Consider $f: \mathbb{R} \to \mathbb{R}$ s.t. $A = \mathbb{R}$ and $f(x) = 0\ \forall x \in \mathbb{R}$

then $f(A) = 0$ which is not open

(b) $\boxed{\text{False}}$

Consider $f: \mathbb{R} \to \mathbb{R}$ s.t. $A = \mathbb{R}$ ~~[illegible]~~ and $f(x) = \frac{1}{x^2}\ \forall x \in \mathbb{R}$

then $f(A) =$ ~~[illegible]~~ $(0, \infty)$

which is not closed since the sequence $\frac{1}{2^i}$ as $i \to \infty$ goes to 0 which is not in $f(A)$

(c) $\boxed{\text{False}}$

bounded by -5 and 5

Consider $f: \mathbb{R} \to \mathbb{R}$ s.t. $A = (-5, 0) \cup (0, 5)$ and $f(x) = \frac{1}{x}\ \forall x \in \mathbb{R}$

then $f(A) =$ ~~[illegible]~~ $(-\infty, -0.2) \cup (0.2, \infty)$ which is unbounded

(d) ~~[illegible]~~ $\boxed{\text{True}}$ Let ~~[illegible]~~ U be an open cover of $f(A)$. Consider U' to be the open cover of ~~[illegible]~~ A inverting this function. U' has a finite subcover, so we can map those sets back ~~[illegible]~~ to get a finite subcover of U.

(e) $\boxed{\text{True}}$

Assume for contradiction that $f(A)$ is not connected.

Then $f(A)$ can be written as union of 2 disjoint sets.

Since f is continuous, their pre image also can't be

one full ~~set~~ subset but rather the union of 2 disjoint sets.

This implies A is not connected $\Rightarrow$ contradiction

(3)

Since $[0,1]$ is compact and $\mathbb{R}$ is not compact

such an f can't exist by #2 if we let $A = [0,1]$

$f(A) = \mathbb{R}$.