Ⓒ For each sequence, give its lim sup and lim inf.

① For $a_n = (-1)^n$, the lim sup = 1 and lim inf = -1.
② For $b_n = \frac{1}{n}$ the lim sup = lim inf = 0
③ For $c_n = n^2$ the lim sup = lim inf = $+\infty$
④ F $d_n = \frac{6n+4}{7n-3}$ the lim sup =

ⓓ Which of the sequences converges? Diverges to $+\infty$ or $-\infty$?
$a_n$ diverges, $b_n$ converges to 0, $c_n$ diverges to $+\infty$, $d_n$

ⓔ Which of the sequences is bounded?
$a_n$ is bounded between -1 and 1. $b_n$ is bounded b/c it converges. (
unbounded. $d_n$ is

**11.2** Consider the sequences defined as follows:

$a_n = (-1)^n$, $b_n = \frac{1}{n}$, $c_n = n^2$, $d_n = \frac{6n+4}{7n-3}$

(a) For each sequence, give an example of a monotone subsequence.

① For $a_n = (-1)^n$, an example of a monotone subsequence would be
- $a_k = (-1)^{2k}$

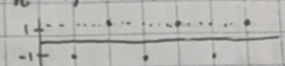
② For $b_n = \frac{1}{n}$, an example of a monotone subsequence would be $b_n = \frac{1}{n}$ since this is already decreasing.

③ For $c_n = n^2$, an example of a monotone subsequence would be $c_n = n^2$ since this is already monotone increasing.

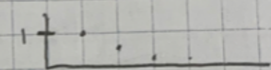
④ For $d_n = \frac{6n+4}{7n-3}$, an example of a monotone subsequence would be

SCRATCH WORK

① $a_n = -1, 1, -1, 1, -1, 1 \ldots$



② $b_n = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$



④ $d_n = \underset{n=1}{\frac{10}{4}}, \underset{n=2}{\frac{16}{11}}, \underset{n=3}{\frac{22}{18}}, \underset{n=4}{\frac{28}{25}}, \underset{n=5}{\frac{34}{32}}, \underset{n=6}{\frac{40}{39}}, \underset{n=7}{\frac{46}{46}}$

$2.5 \; \sim 1.45 \; \sim 1.22 \; \sim 1.12 \; \sim 1.06 \; \sim 1.02 \; 1$

$\underset{n=8}{\frac{52}{53}}, \underset{n=9}{\frac{58}{61}}, \underset{n=10}{\frac{64}{68}}$ observe that it starts to decrease at $n=7$.

(b) For each sequence, give its set of subsequential limits.

① For $a_n = (-1)^n$, its set of subsequential limits is $\{-1, 1\}$.

② For $b_n = \frac{1}{n}$, its set of subsequential limits is $\{0\}$

③ For $c_n = n^2$ its set of subsequential limits is $\{+\infty\}$

- ④ For $d_n = \frac{6n+4}{7n-3}$ its set of subsequential limits is

# MATH 104 HW 3

## PART 1 - ROSS 10.6, 11.2, 11.3, & 11.5

$\frac{1}{2^n}$

**10.6** (a) Let $(s_n)$ be a sequence such that $|s_{n+1} - s_n| < 2^{-n}$ for all $n \in \mathbb{N}$. Prove $(s_n)$ is a Cauchy sequence and hence a convergent sequence.

- ☆ A seq is a Cauchy seq if for each $\epsilon > 0$, $\exists N$ s.t. $m, n > N$ implies $|s_n - s_m| < \epsilon$
- ☆ $n=1 \Rightarrow |s_2 - s_1| < \frac{1}{2}$, $n=2 \Rightarrow |s_3 - s_2| < \frac{1}{4}$, $n=3 \Rightarrow |s_4 - s_3| < \frac{1}{8}$
  Observe that $|s_{n+1} - s_n|$ is getting smaller, but so is the epsilon window.
- ☆ $s_{n+1} < 2^{-n} + s_n \Rightarrow$ the next term in the sequence is less than the one previous it.
- ★ $s(n) = \frac{1}{2^n}$ $n=1$: $|s_2 - s_1| < \frac{1}{2}$, $s(1) = \frac{1}{2}$, $s(2) = \frac{1}{4}$, $s(3) = \frac{1}{8}$
  $= \left|\frac{1}{4} - \frac{1}{2}\right| < \frac{1}{2}$
  $n=2$: $|s_3 - s_2| < \frac{1}{2^2} = \left|\frac{1}{8} - \frac{1}{4}\right| < \frac{1}{4}$ $\quad = \left|\frac{1}{2^n} + \frac{-1}{2^m}\right| < \epsilon$

$n=1: |s_2 - s_1| < \frac{1}{2}$

$n=2: |s_3 - s_2| < \frac{1}{4}$

$n=3: |s_4 - s_3| < \frac{1}{8}$

$r^n(1 + r^1 + r^2 + \dots)$

$r^n + r^{n+1} + r^{n+2} + \dots$

$(1-r)(1 + r + r^2 + \dots) = 1$

$1 + r + r^2 + \dots$

$-r - r^2 + \dots$

$\epsilon = \frac{1}{2}$ find $N$

$n \geq m, > 1$

$|s_n - s_m| < \frac{1}{2}$

$\frac{1}{2} + \frac{1}{4} < \frac{1}{2}$ fix $n=2$

$|s_2 - s_{10}|$ $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4$

$s_2 = 1 + \frac{1}{2}$

$s_{10} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \cdots \frac{1}{2^9} < \frac{1}{2}$

$n=2$ $r^n + r^{n+1} + r^{n+2} \dots$

$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 \dots$ $\quad \frac{1}{1-r}$

$\left(\frac{1}{n}\right)^2\left(1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \dots\right) = \left(\frac{1}{2}\right)^2\left(1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \dots\right)$

$\left(\frac{1}{n}\right)^2\left(\frac{1}{1 - \frac{1}{n}}\right) < \frac{1}{2} = \left(\frac{1}{2}\right)^2\left(\frac{1}{1 - \frac{1}{2}}\right) = \frac{1}{4} \cdot 2 = \frac{1}{2}$