Homework 11

1. 34.2.

(a) $\lim_{x\to 0} \frac{1}{x} \int_0^x e^{t^2}\, \cancel{dt}$

So here we have.

$$\lim_{x\to 0} \frac{1}{x}\int_0^x e^{t^2}\,dt = \lim_{x\to 0} \frac{d}{dx}\left(\frac{1}{x}\int_0^x e^{t^2}\,dt\right).$$

$$= \frac{d/dx\left(\int_0^x e^{t^2}\,dt\right)}{d/dx\,(x)}$$

$$= \frac{e^{x^2}}{1}$$

$$\lim_{x\to 0} \frac{e^{x^2}}{1} = 1.$$

So $\lim_{x\to 0} \frac{1}{x}\int_0^x e^{t^2}\,dt = 1.$

Answer $= 1.$

(b). $\lim_{h \to 0} \frac{1}{h} \int_3^{3+h} e^{t^2} dt.$

$$\lim_{h\to 0} \frac{1}{h}\int_3^{3+h} e^{t^2}\,dt = \lim_{h \to 0} \frac{d}{dh}\left(\frac{1}{h}\int_3^{3+h} e^{t^2}\,dt\right).$$

$$= \frac{d/dh\left(\int_3^{3+h} e^{t^2}\,dt\right)}{d/dh\,(h)}$$

$$= d/dh\left(\int_3^{3+h} e^{t^2}\,dt\right). = d/dh\left(\int_0^h e^{(t+3)^2}\,dt.\right)$$

$$= e^{(h+3)^2}.$$

so we get $\frac{e^{(h+3)^2}}{1}$

$$\lim_{h\to 0} \frac{e^{(h+3)^2}}{1} = e^{(0+3)^2} = e^9.$$

so $\lim_{h\to 0} \frac{1}{h}\int_3^{3+h} e^{t^2}\,dt = e^9.$

Answer $= e^9$.

2. 3hw 5 d

$$F(x) = \int_{x-1}^{x+1} f(t)\, dt.$$

Here we need to show that F is differentiable and we need to find F'.

We know by the Fundamental Theorem of Calculus II that.

If f is continuous at x_0 in (a, b), then. F is differentiable at x_0.

So here we can assume that f is a continuous function in $\mathbb{R}$.

So we can say that.

$$h(x) = \int_{1}^{x} f(t).\, dt$$

Now we can see that

$$F(x) = \int_{x-1}^{x+1} f(t)\, dt.$$

$$= \int_1^{x+1} f(t)dt - \int_1^{x-1} f(t).dt$$

we know.

$$h(x+1) = \int_1^{x+1} f(t)dt$$

$$h(x-1) = \int_1^{x-1} f(t)\,dt.$$

so $F(x) = h(x+1) - h(x-1)$.

Here we know h is differentiable

so as we know h and $x+1$ and $x-1$ are differentiable, we can see that $F(x)$ is a differentiable function

now we need to compute.

$$F(x) = h(x+1) - h(x-1)$$

$$F'(x) = (h(x+1) - h(x-1))'$$

$$= h'(x+1) - h'(x-1)$$

$$h'(x+1) = f(x+1) - f(1)$$

$$h'(x-1) = f(x-1) - f(1)$$

$$= f(x+1) - f(1) - f(x-1) + f(1)$$

$= f(x+1) - f(x-1).$

so $F'(x) = f(x+1) - f(x-1)$

we have found this value.

Answer = we have shown both statements

3. 34.7.

we need to find $\int_0^1 x\sqrt{1-x^2}\,dx$.

we have the formula

$$\int_a^b f \circ u(x)\, u'(x)\, dx = \int_{u(a)}^{u(b)} f(u)\, du.$$

So here we can say that

$u(x) = 1-x^2.$

$u'(x) = -2x\, dx.$

$f(x) = \sqrt{x}$

$f(u) = \sqrt{u}.$

so we have.

$$\int_a^b f\circ u \cdot u'(x)\,dx = \int_{u(0)}^{u(1)} \sqrt{u}\cdot \frac{du}{-2x}\cdot x\,dx$$

$$= \int_1^0 \sqrt{u}\cdot -\frac{1}{2}\,du.$$

$$= -\frac{1}{2}\int_1^0 \sqrt{u}\,du$$

$$= \frac{1}{2}\int_0^1 \sqrt{u}\,du.$$

$$= \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right]_0^1 = \frac{1}{3/2} = \frac{2}{3}.$$

$$\frac{1}{2}\cdot\frac{2}{3} = \frac{1}{3}.$$

So. $\int_0^1 x\sqrt{1-x^2}\,dx = \frac{1}{3}.$

Answer $= \frac{1}{3}$