Huynh, Dylan

f(A) is closed

Hw 7

[0, 1] → [0, 1) → not closed

[0, ∞) → closed

2. a) A is open → f(A) is open

False since if $f(x) = 1$, then f is a continuous mapping but $f(A) = \{1\}$ which is not open since no open ball is contained

b) if A is closed → f(A) is closed

False since suppose we have $f: \mathbb{R} \to \mathbb{R}$ w/ $f(x) = 2^{-x}$. If $A = [0, \infty)$ which is closed since the limit of any seq in A is in A, but $f(A) = [0, 1)$, which is not closed since the limit of an increasing seq in f(A) can be 1 which isn't in [0, 1), so it's not closed.

c) If A is bounded, then f(A) is bounded

False since suppose we have $f: \mathbb{R} \to \mathbb{R}$ w/ $f(x) = \tan(x)$. If $A = (0, \frac{\pi}{2})$, which is clearly bounded, but $f(A) = (0, \infty)$, which is unbounded

d) If A is compact, then f(A) is compact

If A is compact, then every seq a_n in A has a subseq (a_{n_k}) converging to a limit in A. Then, if we take any seq b_n in f(A) where $\exists\, a_n \in A$ s.t. $f(a_{n_k}) = b_{n_k}$, any subseq b_{n_k} must converge to $f(a)$ where $a_{n_k} \to a$ since continuous preserve the convergence of sequences, so f(A) is compact

e) If A is connected, then f(A) is connected

$X \times Y = \bigcup U_\alpha$, want finite subcollection s.t. still covered

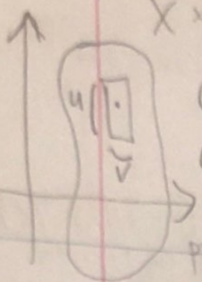
$X = \bigcup_{\alpha=1} U_\alpha$, we can find finite subcover

Given an arbitrary open cover

And that $f^{-1}(B), f^{-1}(C)$ are non-empty, open, disjoint

True since we can try to prove the contrapositive. If we know $f(A)$ is disconnected, $\exists$ non-empty, disjoint open sets B, C s.t. $f(A) = B \sqcup C$. We WTS that $A = f^{-1}(B) \sqcup f^{-1}(C)$. Since f is continuous, $f^{-1}(B)$ and $f^{-1}(C)$ are open. $f^{-1}(B)$ and $f^{-1}(C)$ are disjoint since if $x \in f^{-1}(B) \cap f^{-1}(C)$ $f(x) \in B \cap C$ which is impossible since B, C are disjoint, so thus A is disconnected.

3. Prove: there's no continuous map $f: [0,1] \to \mathbb{R}$ that f is surjective. (that all things in the codomain are mapped to by $x \in [0,1]$)
We know $[0,1]$ is compact since it's closed and bounded. Thus, since f is a cont map, the set $f([0,1])$ which is $\mathbb{R}$ if f is surjective, But $\mathbb{R}$ would be compact since f is a continuous map, a contradiction since $\mathbb{R}$ is not compact since it's unbounded, by Heine-Borel



$X \times Y$, we can choose arbitrary ^slice $Y \times \{x_0\}$ s.t. of ~~the~~ an open cover covering $Y \times \{x_0\}$ can have a finite subcovering since Y is compact and since $Y \to Y \times \{x_0\}$ continous, $Y \times \{x_0\}$ compact → can find open set within each finite subcover $U(x_0)$ = open neighborhood that is an open covering of X

product of open sets is open set in product

1. Given $X \times Y$, we can construct ^an open cover $\mathcal{U} = \bigcup_{\alpha \in I} U_\alpha$. We can then choose an arbitrary slice $Y \times \{x_0\}$, $x_0 \in X$ s.t. an open cover covering $Y \times \{x_0\}$ has a finite subcover since we can establish a continuous map $f: Y \to Y \times \{x_0\}$, so $Y \times \{x_0\}$ is compact. We then know that for each set in the finite subcovering, we have $U(x_0)$ an open neighborhood that when intersected with X, we have an open subset in X. Taking the union of all infinite slices we have an open covering of X, and since X is compact we can thus find finite subcover that covers X, so thus $X \times Y$ are open cover compact.