Hw 10 Math 104 Huynh, Dylan

$|\sin^8(e^x)| < |1|$

33.5)

$$\left|\int_{-2\pi}^{2\pi} x^2 \sin^8(e^x)\,dx\right| \le \int_{-2\pi}^{2\pi} \left|x^2 \sin^8(e^x)\right| dx \le \int_{-2\pi}^{2\pi} |x^2 \cdot 1|\,dx$$
$$= \int_{-2\pi}^{2\pi} x^2\,dx = \frac{x^3}{3}\Big]_{-2\pi}^{2\pi}$$
$$= \frac{16\pi^3}{3}$$

33.4) $f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ -1 & \text{if } x \in \mathbb{R}\setminus\mathbb{Q} \end{cases}$

$f(x)$ isn't integrable since $U(f,P) = \sum_{k=1}^{n} M(f,[t_{k-1},t_k])\cdot(t_k - t_{k-1}) = b-a$

$L(f,P) = \sum_{k=1}^{n} a-b$, so $L(f) \ne U(f)$

$M(|f|,[t_{k-1},t_k]) = m(|f|,[t_{k-1},t_k])$

$|f(x)|$ is integrable since $U(f,P) = L(f,P) = \sum_{k=1}^{n} 1\cdot(t_k - t_{k-1}) = b-a$

33.7) a) WTS: $U(f^2,P) - L(f^2,P) \le 2B[U(f,P) - L(f,P)]$ where $B>0$ s.t. $|f(x)| \le B$

$$U(f^2,P) - L(f^2,P) = \sum_{k=1}^{n} \left(M(f^2,[t_{k-1},t_k]) - m(f^2,[t_{k-1},t_k])\right)\cdot(t_k - t_{k-1})$$

$f^2(y)$

$\sup(\{f^2(x) - f^2(y) : x,y \in [t_{k-1},t_k]\})$

$\sup(\{|f(x)+f(y)|\,|f(x)-f(y)| : x,y \in [t_{k-1},t_k]\})$

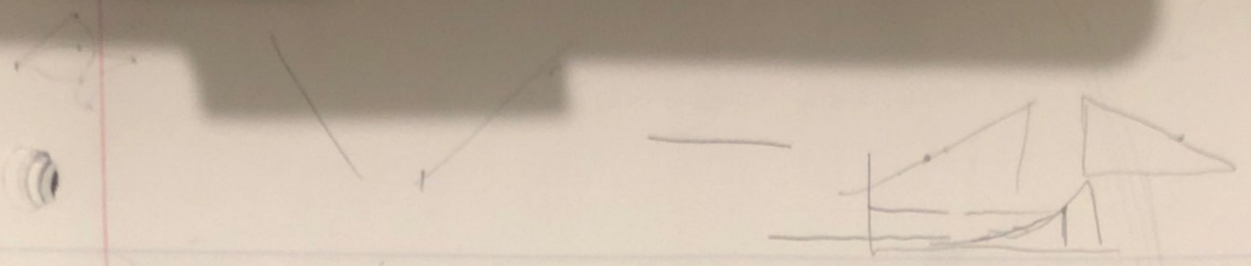
Since $|f(x)+f(y)| \le 2B$ $\quad \le 2B \sup(\{f(x)-f(y)\})\cdot(t_k - t_{k-1})$

$\le 2B\,[U(f,P) - L(f,P)]$

b) if f is integrable on $[a,b]$, then $\exists P$ s.t. $U(f,P) - L(f,P) < \varepsilon_0 = \frac{\varepsilon}{2B}$ $\forall \varepsilon > 0$

We know that $U(f^2,P) - L(f^2,P) \le 2B[U(f,P) - L(f,P)] < \varepsilon$, so

therefore $U(f^2,P) - L(f^2,P) < \varepsilon$, so $U(f^2) = L(f^2)$, so f^2 is integrable on $[a,b]$.



3.13) $\int_a^b f = \int_a^b g$ Since both are continuous, we can use IVT on $h(x) = f(x) - g(x)$. Since we know $\int_a^b f - g = 0 = \int_a^b h$. This means there exists an $x \in (a,b)$ s.t. $h(x) = f(x) - g(x) = \int_a^b h = 0$, so $\exists x$ s.t. $f(x) = g(x)$

35.4) $F(t) = \sin t$ for $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

a) $\int_0^{\pi/2} x\, dF(x) = \int_0^{\frac{\pi}{2}} x \cdot \frac{dF_x}{dx}\, dx = \int_0^{\frac{\pi}{2}} \underset{u}{x} \underset{dv}{\cos x\, dx}$

$= x \cdot \sin x - \int_0^{\frac{\pi}{2}} \sin x\, dx$

$= \frac{\pi}{2} \cdot \sin\frac{\pi}{2} + \cos x \Big]_0^{\frac{\pi}{2}}$

$= \boxed{\frac{\pi}{2} - 1}$

b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos x\, dx = x \cdot \sin x \Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x\, dx$

$= \frac{\pi}{2} \sin\frac{\pi}{2} + \frac{\pi}{2} \sin -\frac{\pi}{2} - 2$

$= \boxed{-2}$

35.9) f is cont on $[a,b]$, we know if we have constant bounds m, M s.t.

$m = \inf(\{f(x) : x \in [a,b]\}) \le f(x) \le \sup(\{f(x) : x \in [a,b]\}) = M$, so

$\int_a^b m\, dF = m \int_a^b dF \le \int_a^b f\, dF \le M \int_a^b dF$

$= m \int_a^b \frac{dF}{dx}\, dx \le \int_a^b f\, dF \le M \int_a^b \frac{dF}{dx}\, dx$

$= m \frac{(F(b) - F(a))}{F(b) - F(a)} \le \frac{\int_a^b f\, dF}{F(b) - F(a)} \le M \frac{(F(b) - F(a))}{F(b) - F(a)}$.

By IVT since $m \le \frac{\int_a^b f\, dF}{F(b) - F(a)} \le M$, $\exists x \in [a,b]$ s.t. we can reach any value between m and M with $f(x)$, so $\exists x$ s.t. $f(x) = \int_a^b f\, dF / (F(b) - F(a)) \to f(x) \cdot (F(b) - F(a)) = \int_a^b f\, dF$