$\cancel{f}x\,dx$ $\quad F = \int_0^x e^{t^2}$ $\quad f(t) = e^{t^2}$

$fu\,du$ $\quad F' = e^{x_0^2}$ $\quad \int e^{t^2} =$

Huynh, Dylan

Math 104 Hw

34.2 a) $\lim_{x\to 0} \frac{1}{x}\int_0^x e^{t^2}\,dt = \lim_{x\to 0} \frac{e^{x^2}}{1} = 1$

b) $\lim_{h\to 0} \frac{1}{h}\int_3^{3+h} e^{t^2}\,dt = \lim_{h\to 0} \frac{e^{(3+h)^2}}{1} = e^9$

34.5 $F(x) = \int_{x-1}^{x+1} f(t)\,dt$ for $x \in \mathbb{R}$ $= \int_0^{x+1} f(t)\,dt - \int_0^{x-1} f(t)\,dt$

Since f is a continuous function, the F is differentiable at $x+1, x-1 \in \mathbb{R}$, and thus by FTC, we have $F'(x) = f(x+1) - f(x-1)$

34.7. $\int_0^1 x\sqrt{1-x^2}\,dx \quad u(x) = 1-x^2$

$= -\frac{1}{2}\int_1^0 \sqrt{u}\,\frac{du}{dx}\cdot dx$

$= -\frac{1}{2}\int_1^0 \sqrt{u}\,du = \frac{1}{2}\int_0^1 \sqrt{u}\,du = \frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_0^1 = \frac{2}{3}\cdot\frac{1}{2} = \boxed{\frac{1}{3}}$

Rudin 15. $\left|\int_a^b fg\,d\alpha\right| \le \left\{\int_a^b |f|^p\,d\alpha\right\}^{1/p}\left\{\int_a^b |g|^q\,d\alpha\right\}^{1/q}$

$\int_a^b f^2(x)\,dx = 1 \quad f(a) = f(b) = 0 \qquad u(x) = x \quad v(x) = f^2(x)$

$\int_a^b x f'(x) f(x) = \frac{x\cdot f^2(x)}{2}\Big]_a^b - \int_a^b \frac{f^2(x)}{2} =$

$= \frac{bf^2(b) - af^2(a)}{2} - \frac{f^2(x)}{2} = -\frac{1}{2}$

$\left(\int_a^b f'(x)\cdot x f(x)\,d\alpha \le \left\{\int_a^b f'(x)^2\,d\alpha\right\}^{\frac{1}{2}}\left\{\int_a^b (xf(x))^2\,d\alpha\right\}^{\frac{1}{2}}\right)^2$

$-\frac{1}{2} \le$

$\frac{1}{4} \le$