Homework 3

Huynh, Dylan

10.6 a) $|s_{n+1} - s_n| < 2^{-n}$ for all $n \in \mathbb{N}$. We can construct ε that $\frac{\varepsilon}{2} > 2^{-n} > 0$ and for all ε $|s_{n+1} - s_n| < 2^{-n} < \frac{\varepsilon}{2}$. WLOG, we have $m \geq n$ where $m = n + k$ for $k \in \mathbb{N}$. We have:

$$|s_m - s_n| = |s_{n+k} - s_n| = |(s_{n+k} - s_{n+k-1}) + (s_{n+k-1} - \ldots + (s_{n+1} - s_n)|$$
$$< |s_{n+k} - s_{n+k+1}| \ldots + |s_{n+1} - s_n|$$
$$< 2^{-(n+k+1)} + \ldots 2^{n} = 2^{-n}(2^{-k-1} + \ldots 2^{0}) < 2^{-n} \cdot 2 = 2^{-n+1} < \varepsilon, \text{ so}$$

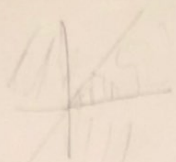
$|s_m - s_n| < \varepsilon$, meaning s_n is Cauchy

b) Result is not true if $|s_{n+1} - s_n| < \frac{1}{n}$ since our proof utilized the fact that $(2^{-k-1} + 2^{-k-2} \ldots + 2^{0}) < 2$ (ie. it's bounded), which is not true for $(1^{-k-1} + 1^{-k-2} \ldots 1) = k$ which diverges as $k \to \infty$, so the distance between s_m and s_n can grow infinitely large

$\frac{10}{4}$

11.2.

	$a_n = (-1)^n$	$b_n = \frac{1}{n}$	$c_n = n^2$	$d_n = \frac{6n+4}{7n-3}$
a)	constant sequence $a'_n = 1$ (a_n where $n = 2k$, $k \in \mathbb{Z}$)	b_n already is a monotone decreasing subseq (for $n \geq 1$)	c_n already is monotone increasing subseq (for $n \geq 1$)	d_n for $n \geq 1$ is monotone decreasing
b)	$\{-1, 1\}$	$\{0\}$	$\{+\infty\}$	$\{6/7\}$
c)	$\limsup(a_n) = 1$ $\liminf(a_n) = -1$	$\limsup(b_n) = 0$ $\liminf(b_n) = 0$	$\limsup(c_n) = +\infty$ $\liminf(c_n) = +\infty$	$\limsup(d_n) = 6/7$ $\liminf(d_n) = 6/7$
d)	diverges	converges to 0	diverge to $+\infty$	converge to $6/7$
e)	bounded	bounded since converges	not bounded, diverges to $+\infty$	bounded since converges



11.3

	$s_n = \cos\left(\frac{n\pi}{3}\right)$	$t_n = \frac{3}{4n+1}$	$u_n = \left(-\frac{1}{2}\right)^n$	$v_n = (-1)^n + \frac{1}{n}$
a)	$s_n' = 1$, (s_n where $n = 6k$ for $k \in \mathbb{Z}$)	t_n for all $n \geq 1$	u_n for $n = 2k$ where $k \in \mathbb{Z}$	v_n for $n = 2k$ where $k \in \mathbb{Z}$
b)	{All reals between and equal to -1 and 1}	$\{0\}$	$\{0\}$	$\{-1, 1\}$
c)	lim sup: 1 lim inf: -1	0 0	0 0	1 -1
d)	diverges	converges to 0	converges to 0	diverges
e)	bounded	bounded	bounded	bounded

11.5. a) Set of subsequential limits S are all rationals between and including 0 and 1. This is because for any rational r between $[0,1]$, we know that there exist an infinite amount of rationals ~~between~~ with ε distance from $\varepsilon > 0$, so we have $|q_n - r| < \varepsilon$ where q_n are any of the infinite rationals ~~satisfy~~ with ε distance. Thus, the set of valid q_n is infinite, so all r are subsequential limits

b) $\liminf q_n = 0$

$\limsup q_n = 1$

2. Lim sup is basically the sup of all subsequences as you approach the tail end of a sequence. Meanwhile, sup is the least upper bound of the entire set, not just the tail end. Something that's counter intuitive about limsup is it disregards everything except the tail end of a sequence. So if you have a monotonic decreasing sequence, the lim sup will be the greatest lower bound of the sequence while the sup is the first term (least upper bound)