Homework 4

Ross §34

2) (a) $\lim_{x\to 0} \frac{1}{x}\int_0^x e^{t^2}\,dt$, $F(x) = \int_0^x e^{t^2}\,dt$

by FTC II, $\lim_{x\to 0} \frac{F(x)}{x} = F'(0) = e^{x^2}\Big|_0 = e^0 = 1$ //

(l'hopital)

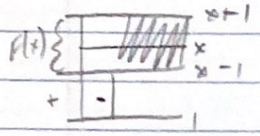
(b) $\lim_{h\to 0} \frac{1}{h}\int_3^{3+h} e^{t^2}\,dt$, $F(x) = \int_3^x e^{t^2}\,dt$

$\lim_{h\to 0} \frac{F(h+3) - F(3)}{h} = F'(3) = e^{3^2} = e^9$ //

5) f cont on $\mathbb{R}$ s.t. $F(x) = \int_{x-1}^{x+1} f(t)\,dt$ for $x \in \mathbb{R}$

Show F is diff. on $\mathbb{R}$ & compute F'.

let $g(x) = \int_1^x f(t)\,dt \Rightarrow g'(x) = f(x)$



$$F(x) = \int_{x-1}^{x+1} f(t)\,dt = \int_{x-1}^{1} f(t)\,dt + \int_1^{x+1} f(t)\,dt$$

$$= \int_1^{x+1} f(t)\,dt - \int_1^{x-1} f(t)\,dt = g(x+1) - g(x-1)$$

$F(x)$ is the sum of diff. fnes $\Rightarrow F$ is diff on $\mathbb{R}$.

$F'(x) = g'(x+1) - g'(x-1) = f(x+1) - f(x-1)$ □

7) $\int_0^1 x\sqrt{1-x^2}\,dx = \frac{-1}{2}\int_1^0 \sqrt{u}\,du = \frac{1}{2}\int_0^1 \sqrt{u}\,du = \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right]_0^1$

$u = 1-x^2$

$du = -2x$

$= \frac{1}{2}\left[\frac{2}{3}\right] = \frac{1}{3}$