

# HW 11

15. a)  $\int f g \leq \left( \int f^p \right)^{\frac{1}{p}} \left( \int |g|^q \right)^{\frac{1}{q}}$

$\int_a^b x f(x) f'(x)$

$\frac{x f(x) \cdot f'(x)}{f(x) + x f'(x)} \quad f(x)$

2 y =

$x f^2(x) - \left( \int x f'(x) f(x) dx + \int f^2(x) dx \right)$

b)  $\int (f'(x))^2 dx \left( \int x f(x)^2 dx \right)^{\frac{1}{2}} \geq \int f'(x) x f(x) dx$

$= \frac{-1}{2}$   
 $= \frac{1}{4}$

Extra:  $\alpha(x)$  is cont. at  $x=0$   $\forall \epsilon > 0, \exists \delta > 0$  s.t.  $|x| < \delta \Rightarrow |\alpha(x) - \alpha(0)| < \epsilon$

if  $x=0$ , trivially true  $< (\frac{1}{2})^{\frac{1}{n}-1} = \epsilon$

$x \neq 0$ ,  $\exists n$  s.t.  $\sum_{k=0}^n (\frac{1}{2})^k < \epsilon$

$n = \lceil \log_{\frac{1}{2}} \epsilon \rceil$

$$34.2 \text{ a) L'Hopital's: } \lim_{x \rightarrow 0} \frac{e^{x^2}}{1} = \boxed{1}$$

$$\text{b) } = \lim_{x \rightarrow 3} e^{x^2} = F'(3) = \boxed{e^9}$$

$$34.5 \quad \lim_{y \rightarrow y} \frac{F(x) - F(y)}{-y} \quad \lim_{h \rightarrow 0} \frac{F(x+h) - F(x)}{h}$$

$$F(x) = \int_0^{x+1} f dt + \int_{x-1}^0 f dt$$

( $x+y+z$ ) which is differentiable,

$$F'(x) = \boxed{f(x+1) - f(x-1)}$$

$$34.7) = \int -\frac{1}{2} \sqrt{u} du \quad u = 1-x^2$$

$$= -\frac{1}{3} u^{3/2} \Big|_0^1$$

$$= \boxed{\frac{1}{3}}$$