HW 5

13.3 a) $d(x,y) \geq 0$ because $|x_j - y_j| \geq 0$.
$d(x,y) = 0 \Rightarrow x = y$. $|x_j - y_j| \geq 0$.
$\sup\{\cdots\} = 0 \Rightarrow |y_j - y_j| = 0 \ \forall j \in \mathbb{N}$
$x_j = y_j \ \therefore x = y$
$x = y \Rightarrow d(x,y) = 0$, Since $x_j = y_j \ \forall j$, $\sup\{\cdots\} = 0$
Triangle ineq: $d(x,z) \geq d(x,y) + d(y,z)$
$|x - z| \leq |x - y| + |y - z|$
$\sup\{|x - z|\} \leq |x - y| + |y - z|$
$d(x,z) \leq d(x,y) + d(y,z)$
b) Yes, it is a metric.

13.5 a) Let $x \in LHS$. $x \notin U \ \forall U \in \mathcal{U}$. ~~So~~ so $x \in S \setminus U \{U\}$
$\therefore x \in RHS$.
Let $x \in RHS$. " " $\therefore x \in LHS$.
b) $\bigcap\{S_i\}$ is closed, call this A.
Let $p_n \to p$ be a sequence in A. $p_n \in S_i \ \forall i$
Since S_i is closed, $p_n \to p$ and $p \in S_i \ \forall i$.
Since $p \in S_i \ \forall i$, $p \in A$, thus A is closed.

4. Let $S_1 = \overline{S}$. Then $\forall E$ s.t. E is closed and
$S \subset E$, $\bigcap E = S_1$
E is closed iff $E = \overline{E}$.
Let $S_1 = \overline{S}$, WTS S_1 is closed.
Let $p_n \to p$, $p_n \in S_1$. To show $p \in S_1$,
we can show p is a limit of a sequence in S.
Let $q_n \in S$. Every p_n is a limit of a sequence
$\{q_n\}_i \in S$ $(q_{n,i}) \to p_n$.
$q_n = (q_{n,i})_n$, as $n \to \infty$, $q_n \to p$.
$\therefore p \in S$, so p is in S_1.
Therefore, since S_1 is closed, $S_1 = \overline{S_1}$.

13.7. If an open set in $\mathbb{R}$ ~~is~~ $S - 13$
only one interval, this is trivially
true. Otherwise, it must be
composed of multiple intervals. Either
way, this holds true.

5. W.T.S $\bar{S} = \cap \{U \in X \mid S \in U\}$
$\bar{S} = \{p \in X \mid \exists p_n \in S, p_n \to p\}$
Let U be a closed subset in X that contains S.
Let $p \in \bar{S}$. $\exists p_n \in S$ s.t. $p_n \to p$.
Since $p_n \in S$, $p_n \in U$, and $\therefore p \in U$.