HW 10

33.13) $f(x_1) = \frac{1}{b-a}\int_a^b f(x)\,dx$ and $g(x_2) = \frac{1}{b-a}\int_a^b g$

$\int_a^b f = \int_a^b g$ $\qquad f'(x) = g(x)$

By IVT: $\frac{F(b)-F(a)}{b-a} = f(x)$ for some $x \in (a,b)$

and $\frac{G(b)-G(a)}{b-a} = g(x)$

$f(a) < k < f(b)$

$f(c) = k$

Since $h(x) = f(x) - g(x)$, $\exists x$ $h(x) = 0$ since $\int_a^b h(x) = 0$, by IVT.

33.7) a) Let P be partition.

~~$L(f^2,P) \le U(f,P)^2$~~

$\sum |f(x)^2 - f(y)^2| \cdot (x-y)$

$(f(x) - f(y))(f(x) + f(y))(x-y)$

$\le 2B$

$\le \boxed{2B\,(U(f,P) - L(f,P))}$

b) Let $\varepsilon > 0$. $\exists P$ s.t. $U(f,P) - L(f,P) < \varepsilon$.

$U(f^2,P) - L(f^2,P) \le 2B\,[U(f,P) - L(f,P)]$

$< 2B\varepsilon = \boxed{\varepsilon_1}$

So f^2 is integrable.

33.4) $f(x) = 1 \quad x \in \mathbb{Q}$,

$= -1 \quad x \notin \mathbb{Q}$.

35.4 a) $\int_0^{\pi/2} x \cos x\,dx$ $\qquad -\cos x + x\sin x \Big|_0^{\pi/2}$

$= \frac{\pi}{2} - 1$

b) $\frac{\pi}{2} - (-1) - (-\frac{\pi}{2})(-1) = \frac{\pi}{2} + 1 - \frac{\pi}{2} = \boxed{2}$

35.9 a) $m = \min f$, $M = \max f$ on $[a,b]$

$m \le \frac{\int_a^b f\,dF}{F(b)-F(a)} \le M$

So by IVT,

$\exists x \in (a,b)$ s.t. $f(x) = \boxed{\frac{\int_a^b f\,dF}{F(b)-F(a)}}$