HW 3

10.6, 11.2, 11.3, 11.5

10.6 a) $\forall \varepsilon > 0,\ |s_n - s_m| < \varepsilon \quad \forall\, n, m > N.$

Set $N = \max(1 - \log_2 \varepsilon,\ 0)\quad \forall \varepsilon > 0$

If $\varepsilon < 1$: $|s_n - s_m| = \sum_{i=n}^{m} 2^{-i}$, $N < n < m$

$\leq 2^{-n+1} \leq 2^{-1+\log_2 \varepsilon + 1} = \varepsilon$

So $|s_n - s_m| \leq \varepsilon$.

If $\varepsilon \geq 1$, same logic works as $N = 0$.

b) No.

11.2

	a_n	b_n	c_n	d_n
a)	$n = 2i,\ i \in \mathbb{Z}$	$n \geq 1$	$n \geq 1$	$n \geq 1$
b)	$\{-1, 1\}$	$\{0\}$	$\{+\infty\}$	$\{\frac{6}{7}\}$
c)	l.s. = 1 l.i. = −1	l.s = l.i = 0	l.s = l.i = $+\infty$	l.s = l.i = $\frac{6}{7}$
d)	oscillating, diverges	converges	diverge $+\infty$	converge
e)	bounded	bounded	unbounded	bounded

11.3

	a_n	b_n	c_n	d_n
a)	$n = 2\pi i,\ i \in \mathbb{Z}$	$n \geq 1$	$n = 2i,\ i \in \mathbb{Z}$	$n = 2i,\ i \in \mathbb{Z}$
b)	$\{x \in \mathbb{R},\ \lvert x \rvert \leq 1\}$	$\{0\}$	0	$\{1, -1\}$
c)	l.s = 1 l.i = −1	ls = li = 0	ls = li = 0	ls = 1 li = −1
d)	diverges	converge	converge	diverge
e)	bounded	bound	bounced	bounded

11.5 a) $S = \{x \in \mathbb{Q} \mid 0 \leq x \leq 1\}$

b) $\limsup q_n = 1 \quad \liminf q_n = 0$

2. lim sup is the value which the sups of subsequences of s_n converge to. lim sup being monotonic is a little counterintuitive. limsup changes for decreasing sequences.