HW 4

14.2 a) $\sum \frac{1}{n} - \frac{1}{n^2}$ diverges since $\sum \frac{1}{n}$ diverges and $\sum \frac{1}{n^2}$ is convergent.

b) $\neg \exists N, n > N, \forall \varepsilon > 0 \quad \sum_n^m |s_n| < \varepsilon$.
diverges

c) converges $= \sum \frac{3}{n^2}$

d) $\limsup \frac{n^{\frac{3}{n}}}{3} = \limsup \frac{(n^{\frac{1}{n}})^3}{3} = \lim \frac{(n^{\frac{1}{n}})^3}{3} = \frac{1}{3}$

so convergent by root test

e) $\frac{(n+1)^2}{(n+1)!} \cdot \frac{n!}{n^2} = \frac{(n+1)}{n^2}$ $\lim R = 0$ so conv.

f) convergent by root test.

g) convergent by root test $\limsup \frac{n^{\frac{1}{n}}}{2} = \frac{1}{2} < 1$

14.10. $2^{n-(-1)^n}$ it is the inverse of the other one.

Rudin:

6.a) $= \frac{1}{\sqrt{n+1} + \sqrt{n}} > \frac{1}{2\sqrt{n}}$ which $\boxed{\text{diverges}}$

b) $\frac{1}{n(\sqrt{n+1} + \sqrt{n})} \leq \frac{1}{2n\sqrt{n+n}}$ so converges

c) converges by root test

d)

7. $\sum \frac{a_n}{n\sqrt{a_n}} \leq \frac{a_n}{n\sqrt{a_n}}$ $a_n - 2\frac{\sqrt{a_n}}{n} + \frac{1}{n^2} \geq 0$

$\frac{1}{2}\left(a_n + \frac{1}{n^2}\right)$

$(a-b)(a-b)$

9. a) 1 b) $R = \infty$

12.2 Let $n > m$ $\sup\{\sigma_n : n > M\} \leq \frac{1}{M}(s_1 + \cdots + s_N) + \sup\{s_i : n > N\}$

$M \to \infty$ and $N \to \infty$

$\limsup \sigma_n \leq \limsup s_n$

b) if $\lim s_n$ exists then $\limsup \sigma_n = \liminf s_n$

$\therefore \lim s_n = \lim \sigma_n$.

12.10 $\exists K \quad s_n \le K \quad n > 0$
then $\limsup |s_n| \le K$.

if $\limsup s = L$ then L is also a bound for s_n