HW 6

1. Let a_n be a seq. in X, $b_n \in Y$.
Let $(a_n)_k$ be subseq in X, $(b_n)_k \in Y$
~~For all subse~~ $\exists (a_n)_k \to a$ since X is compact,
and $\exists (b_n)_k \to b$ "
~~Thus~~ For these two subsequences,
all sub-subsequences also converge.
Therefor, take $(a_n)_{k,l}$ and $(b_n)_{n,l}$
as $l \to \infty$. These converge to (a, b).
Therefor $X \times Y$ is compact.

2. E is $\boxed{\text{uncountable}}$. By cantor's diagonalization argument, we can construct a new decimal expansion that is not in E.

Let $S_n \in E$, $S_n \to S$. ASOC $S \notin E$
That means S has a digit in its decimal expansion that isn't 4 or 7.
Since $S_n \to S$, $\forall \varepsilon > 0$, $\exists N > 0$ s.t $\forall n > N$
$|S_n - S| < \varepsilon$. Since S has a non 4/7 decimal digit, $|S_n - S| = k > 0$, and
all $\varepsilon < k$ cannot be reached by $|S_n - S|$.
Therefor, contradiction, $S_n \not\to S$,
so E is a closed subset of $[0,1]$
so E is compact.

3. Can ~~be~~ $\overline{\bigcup_i A_i} \neq \bigcup_i \overline{A_i}$?
Yes. ~~The~~ Let A_i be the ith rational # in an enumeration of rationals.
$\bigcup_i \overline{A_i} = \mathbb{Q}$, but $\overline{\bigcup_i A_i} = \mathbb{R}$.
It is possible to be strict. $\square$

4. The argument fails because the complement of a countable union of open intervals is not necessarily also countable. For example, the Cantor Set is a closed set which is not a countable union of closed intervals, and yet has a countable complement.