HW 7

1. Let O be an open covering of $X \times Y$.
Let O_X and O_Y be the X and Y components of the set O. Since X and Y are compact, there is a finite subcovering of O_X and O_Y. Let
$O_{XY} = \{(x, y) \mid \forall x \in O_X \, \forall y \in O_Y\}$. O_{XY} is a finite subcovering of O. Therefor, O is open ~~cover~~ compact.

2. 1) False. ~~Let~~ $f(x) = $ ~~A~~ $\sin x$ $A = (-2\pi, 2\pi)$
$f(A) = $ ~~(0~~ $[0, \infty)$

2) False, $f(x) = \frac{1}{x}$ $A = [0, 1]$
$f(A) = [1, \infty)$

3) . False. $f(x) = \frac{1}{x}$. $A = [0, 1]$ $f(A = [1, \infty]$.

4) True. Let O be a covering of $f(A)$.
Since f is continuous, ~~A is open~~ the preemage of O in A is open. Set O to be equal to the union of the images of A's finite subcovering. Thus $f(A)$ is compact.

5) ~~Since~~ We cannot write $f(A)$ as a disjoint union of two subsets of $f(A)$. Since A is connected, any two subsets disjoint in $f(A)$ have preimage that is connected, as is the union of these in A. Therefor, they will be connected in $f(A)$.

3. $[0, 1]$ is compact, and since f is continuous, $\mathbb{R}$ is compact if f is possible. But $\mathbb{R}$ is not compact, so contradiction.