HW 6

1. take any point $[a, b] \in [0,1]^2$, then there must be conv. subseq that approach any point in $[0,1]$. Similar to the proof of Conv. $\to$ Cauchy,

$\exists N_1$ s.t. for $n > N_1$, $d([u_n, 0], [u, 0]) < \frac{\epsilon}{2}$

$\exists N_2$ s.t. for $n > N_2$, $d([0, v_n], [0, v]) < \frac{\epsilon}{2}$

take $\bar{N} = \max(N_1, N_2)$, then $\forall n > \bar{N}$, $d([u_n, 0], [u, v]) < d([0, v_n], [0, v]) = \epsilon$

Then for any point in $[0,1]^2$, $[u_n, v_n] \to [u, v]$

2. Assume E is finite, then enumerate $p \in E$: $p_i = \sum_{i=1}^{n} \frac{4}{10^i}$, $p_1 = 0.4$, $p_2 = 0.44$, enumerating all finitely many of p's, it's always possible to construct another p that's not enumerated. $\Rightarrow$ E is not countable.

Still working on compact.

3. Take A to be $A_i = (\frac{1}{i}, 1)$, $B = \bigcup_i A_i = (0, 1)$,

the closure of B is $\bar{B} = [0,1]$, and $\forall i$, $\{0\} \cap [\frac{1}{i}, 1] = \phi$

We have a point in $\bar{B}$ that is not in $\bigcup_i A_i = (0,1)$, hence it's a strict inclusion.

4. $\mathbb{R}$ is closed in $\mathbb{R}$ trivially, but $\mathbb{R}$ is also not countable.

We can't reconstruct $\mathbb{R}$ fully using a union of finite closed intervals.

If we take a finite set of intervals $U = U_i$ $\forall i$ where $U_i = [i, i]$,

then $\bigcup U_i = U_{\max(i)}$, and $\exists (i + e) \in \mathbb{R}$ that is outside of $\bigcup U_i$.

$\Rightarrow$ we can't cover $\mathbb{R}$ using a finite set of closed intervals.