34.2) a) $\lim_{x\to 0} \frac{1}{x} \int_0^x e^{t^2} dt$

Let $F(x) = \int_0^x e^{t^2} dt$

then $F'(a) = \lim_{x\to a} \frac{1}{x-a}\left(\int_0^x e^{t^2} dt - \int_0^a e^{t^2} dt\right)$

so $F'(0) = \lim_{x\to 0} \frac{1}{x}\left(\int_0^x e^{t^2} dt\right)$

By fundamental thm of calculus

$$F'(0) = e^{0^2} = \boxed{1}$$

b) $\lim_{h\to 0} \frac{1}{h} \int_3^{3+h} e^{t^2} dt$

Let $f(x) = \int_0^x e^{t^2} dt$

$$F'(x) = \lim_{h\to 0} \frac{1}{h}\left(\int_0^{x+h} e^{t^2} dt - \int_0^x e^{t^2} dt\right)$$

$$F'(3) = \lim_{h\to 0} \frac{1}{h}\left(\int_0^{3+h} e^{t^2} dt - \int_0^3 e^{t^2} dt\right)$$

$$F'(3) = \lim_{h\to 0} \frac{1}{h} \int_3^{3+h} e^{t^2} dt$$

By fundamental thm of calculus $F'(3) = \boxed{e^9}$

34.5) $\lim_{h\to 0} \frac{1}{h}\left(\int_{x+h-1}^{x+h+1} f(t)\, dt - \int_{x-1}^{x+1} f(t) dt\right)$

$$\lim_{h\to 0} \frac{1}{h}\left(\int_{x+1}^{x+h+1} f(t) dt\right) - \frac{1}{h}\left(\int_{x-1}^{x-1+h} f(t)\, dt\right)$$

Fundamental thm calc

$$= f(x+1) - f(x-1)$$

Thus since lim exists, F is differentiable on $\mathbb{R}$ and $F' = f(x+1) - f(x-1)$

34.7) $\int_0^1 x\sqrt{1-x^2}\,dx$ $\begin{cases} u = 1-x^2 \\ du = -2x\,dx \end{cases}$

$= \int_1^0 \frac{\sqrt{u}}{-2}\,du$

$= \frac{1}{2}\int_0^1 \sqrt{u}\,du$

$= \frac{1}{3}\left[u^{3/2}\right]_0^1$

$= \boxed{\frac{1}{3}}$