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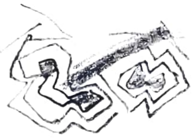
Math Hwk 7



1) For any $\{A_i\}$ s.t. $X \subset \bigcup^{\infty} A_i$ and A_i open $\forall i$

For any $\{B_i\}$ s.t. $Y \subset \bigcup^{\infty} B_i$ and B_i open $\forall i$

$X \subset \bigcup^{N} A_i$, $Y \subset \bigcup^{M} B_i$ since open-cover compact

For any $\{C_i\}$ st $(X \times Y) \subset \bigcup^{\infty} C_i$

Take $\{A_i\}$ to be set of open covers of first coordinate $\{C_i\}$

" $\{B_i\}$ " " " " " " " second " $\{C_i\}$

Then, there exist $\bigcup^{N} A_i$ st $X \subset \bigcup^{N} A_i$ N finite

and " " $\bigcup^{M} B_i$ st $Y \subset \bigcup^{M} B_i$ M finite

Then take the set of covers of $\{C_i\}$ which have $\bigcup^{N} A_i$ as first coordinate, taking only one for each A_i.

Do the same for B_i

Union these two sets and $X \times Y \subset \bigcup^{N+M} \{C_i\}$

and $N+M$ is finite.

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2) a) A open $\Rightarrow$ f(A) open

$\boxed{\text{False}}$ there isn't necesarily a continuous f^{-1}

eg: $f: \mathbb{R} \to \mathbb{R}$

$x \mapsto x^2$

if $A = (-1, 2]$, it is open. But $f(A) = [0, 4]$ is not open.

b) $\boxed{\text{FALSE}}$ Eg) $X = Y = A = \mathbb{R}$

Since $A = X$, A is closed. $f(x): \mathbb{R} \to \mathbb{R}$ continuous

$x \mapsto 2^x$

$f(A) = (0, \infty)$ not closed.

2 c) $\boxed{\text{FALSE}}$ eg) $X = Y = \mathbb{R}$

Let $A = (0,5)$ $f(x) = \ln(x)$ continuous

$f(A)$ not bounded

d) $\boxed{\text{TRUE}}$ For any open cover $\bigcup^{\infty} C_i$ for A, $\exists \bigcup_{1}^{N} C_i$ open cover for A.

Take open covers $\bigcup^{\infty} D_i$ for $f(A)$, since f is a continuous map, $\bigcup^{\infty} f^{-1}(D_i)$ open, and must cover

A if take union of all possible f^{-1}'s. Since A is compact, $\exists \bigcup^{N} f^{-1}(D_i)$ which is open and covers A.

So $\bigcup^{N} f(f^{-1}(D_i))$ covers $f(A)$ so $\bigcup^{N} D_i$ is a finite subcover of $f(A)$. Compact ✓

e) $\boxed{\text{TRUE}}$ Assume $f(A) = P \cup Q$, P, Q disjoint open subsets of $f(A)$

then for $p \in A$ $f(p) \in P$ or $f(p) \in Q$

Call preimage of P, $f^{-1}(P)$ & preimage of q, Q^{-1}

since both P, Q are open & f continuous, $f^{-1}(P)$ & $f^{-1}(Q)$ open

but $f^{-1}(P) \cup f^{-1}(Q) = A$ & $f^{-1}(P) \cap f^{-1}(Q) = \emptyset$

so A is not connected. So assumption false. Contradiction

3) $\mathbb{R}$ is open in $\mathbb{R}$. If f is continuous and maps to all $\mathbb{R}$ then the preimage of f (which here is $[0,1]$) must be open. But $[0,1]$ not open so this f can not exist.