33.4, 33.7, 33.13, 35.4, 35.9a

Nathan Sweet

33.4) $\cancel{f(x)=\begin{cases}\frac{\sin(\frac{1}{x})}{|\sin(\frac{1}{x})|} & x\in(0,1)\\ 1 & x=0\end{cases}}$

$f(x)=\begin{cases}1 & x\in\mathbb{Q}\\ -1 & x\notin\mathbb{Q}\end{cases}$ $f(x)$ not integrable, but $|f(x)|$ integrab

33.7) a) $U(f^2,S)-L(f^2,S)=\underbrace{\sup(f^2(x), x\in S)-\inf(f^2(y), y\in S)}$ for a set $S\in P$

$[f(x)+f(y)]\cdot[f(x)-f(y)]$

$f(x)<B \quad f(y)<B$

$\leq [B+B]\cdot[f(x)-f(y)]$

$U(f^2,P)-L(f^2,P)\leq 2B\cdot[U(f,P)-L(f,P)]$

b) If $\lim_P [U(f,P)-L(f,P)]\to 0$ then $\lim_P[U(f^2,P)-L(f^2,P)]\to 0$

since $2B\cdot 0\to 0$

and $f^2\geq 0$ in real numbers

33.13) Assume $f(x)\neq g(x)$ $\forall x\in[a,b]$. Since $f(x)$ & $g(x)$ continuous, $f(x)-g(x)$ continuous on $[a,b]$. Since $f(x)-g(x)\neq 0$ & continuous, $f(x)-g(x)>0$ $\forall x$ or $f(x)-g(x)<0$ $\forall x$, so $f(x)>g(x)$ $\forall x$ or $f(x)<g(x)$ $\forall x$. Therefore, $\int_a^b f(x)-g(x)>0$ or $\int_a^b f(x)-g(x)<0$ so $\int_a^b f(x)\neq\int_a^b g(x)$. Contradiction.

35.4) a) $\int_0^{\frac{\pi}{2}} x\cos(x)\,dx$

$[x\sin(x)]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\sin(x)\,dx$

$[x\sin(x)+\cos(x)]_0^{\frac{\pi}{2}}$

$\boxed{\frac{\pi}{2}-1}$

b) $[x\sin(x)+\cos(x)]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

$\frac{\pi}{2}+\frac{\pi}{2}=\boxed{\pi}$

35.9a) By MVT on $\int_a^x f(t)\,dt$, for which $f(x)$ is derivative, $F_a(x)=[\int_a^b f(t)dt-\int_a^a f(t)dt]/[b-a]$ for some x

Similarly, $[\int_a^b f(t)dF]=f(x)[F(b)-F(a)]$ for some x