# HW10

Jiayin Lin

## 1 Question 1

Consider $f(x) = 1$ for $x \in \mathbb{Q}$ and $f(x) = -1$ otherwise.
This function is obviously not integrable, but $|f(x)| = 1$ is.

## 2 Question 2

denote

$$U(f^2, p) = \sum_{p \in P} f(x_p)^2 \Delta x_p$$
$$L(f^2, p) = \sum_{p \in P} f(y_p)^2 \Delta x_p$$

$x_p$, $y_p$ are in the interval $p$

$$\begin{aligned} U(f^2, p) - L(f^2, p) &= \sum_{p \in P} (f(x_p)^2 - f(y_p)^2) \Delta x_p \\ &= \sum_{p \in P} (f(x_p) - f(y_p))(f(x_p) + f(y_p)) \Delta x_p \\ &\leq \sum_{p \in P} (f(x_p) - f(y_p)) 2B \Delta x_p \\ &= 2B(U(f, p) - L(f, p)) \end{aligned}$$

thus $f^2$ is obviously integrable because the right side=0

## 3 Question 3

If not, since both are continuous, then $f(x) > g(x)$ without loss of generality.
Then, $\int_a^b f(x)dx = (b - a)f(x_1)$ and $\int_a^b g(x)dx = (b - a)g(x_2)$, but then $g(x_2) = f(x_1)$ contradiction.

## 4 Question 4

$$\int_0^{\frac{\pi}{2}} x d\sin x$$
$$= \int_0^{\frac{\pi}{2}} x\cos x dx$$
$$= \cos\frac{\pi}{2} - \cos 0 + \frac{\pi}{2}\sin\frac{\pi}{2} - 0 = \frac{\pi}{2} - 1$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x d\sin x$$
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x dx$$
$$= \cos\frac{\pi}{2} - \cos(-\frac{\pi}{2}) + \frac{\pi}{2}\sin\frac{\pi}{2} - (-\frac{\pi}{2})\sin(-\frac{\pi}{2}) = 0$$

## 5 Question 5

If not, since f is continuous then $f(x) > \frac{\int_a^b f(x)dF}{F(b)-F(a)}$ without loss of generality.
Since f is integrable, then

$$\begin{aligned} A = \int_a^b f(x)dF &= \sum_0^n f(x_k)(F(x_k^+) - F(x_k^-)) + \sum_1^n M(f, x_{k-1}, x_k)(F(x_k^-) - F(x_{k-1}^+)) \\ &> \sum_0^n \frac{A}{F(b)-F(a)}(F(x_k^+) - F(x_k^-)) + \sum_1^n \frac{A}{F(b)-F(a)}(F(x_k^-) - F(x_{k-1}^+)) \\ &> \frac{A(F(x_n^+) - F(x_0^-))}{F(b)-F(a)} = A \end{aligned}$$

so we have A¿A as contradiction.