$k=2$

27

$7+9+11$ $\quad 3(2)^2+8+4$

$n=3$

$n=2$ $\quad 5+7$

$7+9+11$

$5+7+9$

1.10 $n=1 \Rightarrow 3=3$ ✓

$n=k+1 \Rightarrow (2(k+1)+1)+(2(k+1)+3)+(2(k+1)+5)+\ldots+(4(k+1)-1)$

$= (2k+3)+(2k+5)+\ldots(4k+3) = 3k^2+(4k+3)-(2k+1)+(4k+1)$

$= 3k^2+6k+3 = 3(k+1)^2$

$2k+4 = 6k+3$

$3(k+1)$

1.12a. $n=1$ $\quad a+b = \binom{1}{0}a+\binom{1}{1}b = a+b$

$n=2$ $\quad (a+b)^2 = a^2+2ab+b^2 = \binom{2}{0}a^2+\binom{2}{1}ab+\binom{2}{2}b^2$

$n=3$ $\quad (a+b)^3 = a^3+3a^2b+3ab^2+b^3 = \binom{3}{0}a^3+\binom{3}{1}a^2b+\binom{3}{2}ab^2+\binom{3}{3}b^3$

b. $\binom{n}{k}+\binom{n}{k-1} = \frac{n!}{k!(n-k)!}+\frac{n!}{(k-1)!(n-k+1)!} = \frac{n!(n-k+1)+n!k}{k!(n-k+1)!} =$

$\frac{n!(n+1)}{k!(n-k+1)!} = \binom{n+1}{k}$

c. $(a+b)^{n+1} = (a+b)(a+b)^n = a\left[\binom{n}{0}a^n+\binom{n}{1}a^{n-1}b+\ldots+\binom{n}{n-1}ab^{n-1}+\binom{n}{n}b^n\right]$

$+b\left[\binom{n}{0}a^n+\binom{n}{1}a^{n-1}b+\ldots+\binom{n}{n-1}ab^{n-1}+\binom{n}{n}b^n\right] =$

$\binom{n}{0}a^{n+1}+\underbrace{\left(\binom{n}{1}+\binom{n}{0}\right)}_{}a^nb+\left(\binom{n}{2}+\binom{n}{1}\right)a^{n-1}b^2+\ldots+\left(\binom{n}{n}+\binom{n}{n-1}\right)ab^n+\binom{n}{n}b^{n+1}$

$= a^{n+1}+\binom{n+1}{1}a^nb+\ldots+\binom{n+1}{n}ab^n+b^{n+1}$

by part b

2.1 $x^2-3=0$ $\quad$ candidates $= \{\pm1, \pm3\}$

$x^2-5=0$ $\quad$ " $= \{\pm1, \pm5\}$

$x^2-7=0$ $\quad$ " $= \{\pm1, \pm7\}$

$x^2-24=0$ $\quad$ " $= \{\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24\}$

$x^2-31=0$ $\quad$ " $= \{\pm1, \pm31\}$

2.2 $x^3-2=0$ candidates: $\{\pm1, \pm2\}$

$x^2-5=0$ " : $\{\pm1, \pm5\}$

$x^4-13=0$ " : $\{\pm1, \pm13\}$

2.7a $x=\sqrt{4+2\sqrt3}-\sqrt3$

$x+\sqrt3=\sqrt{4+2\sqrt3}$

$x^2+2\sqrt3x+3=4+2\sqrt3$

$x^2+2\sqrt3\,x-1-2\sqrt3$

$(x+(1+2\sqrt3))(x-1)=0$

$x=1$

b. $x=\sqrt{6+4\sqrt2}-\sqrt2$

$x+\sqrt2=\sqrt{6+4\sqrt2}$

$x^2+2\sqrt2\,x+2=6+4\sqrt2$

$x^2+2\sqrt2x-4-4\sqrt2=0$

$(x-2)(x+2+2\sqrt2)=0$

$x=2$

3.1i. $a+c=b+c$

$(a+c)-c=(b+c)-c$

$a+(c-c)=b+(c-c) \Rightarrow a+0=b+0$

$a=b$

ii. $a\cdot0=a\cdot(0+0)=a\cdot0+a\cdot0$

$0=a\cdot0$ by (i)

iii. $ab+(-a)b=(a+(-a))b=0\cdot b=0=ab+(-(ab))$

$(-a)b=-ab$ from (i)

iv. $(-a)(-b)=-a(-b)$

$=-b(-a)$

$(-a)(-b)+(-a)(b)=(-a)(-b+b)=(-a)\cdot0$

$\Rightarrow 0=-ab+ab=\underbrace{(-a)b+ab}_{\text{by iii}} \Rightarrow (-a)(-b)=ab$ by i

v. $ac=bc \Rightarrow \underbrace{ac-bc=bc-bc}_{\text{by i}} \Rightarrow c(a-b)=0 \Rightarrow c\cdot c^{-1}(a-b)=0\cdot c^{-1}$

$\Rightarrow \underbrace{1\cdot(a-b)=0}_{\text{by ii}} \Rightarrow a-b=0 \Rightarrow a-b+b=b \Rightarrow a=b$

vi. $ab=0 \Rightarrow$ WLOG if $b\neq0$, $a\cdot b\cdot b^{-1}=0\cdot b^{-1} \Rightarrow a=0$ WLOG

3.2.i. $a\le b \Rightarrow a+((-a)+(-b))\le b+((-a)+(-b))$

$\Rightarrow -b\le -a$

ii. $a \le b$ prove $(-c) \ge 0$; $c \le 0 \Rightarrow -c \le -0$ by i

$a(-c) \le b(-c) \Rightarrow -ac \le -bc \Rightarrow bc \le ac$ by i.

iii. $0 \le a,\ 0 \le b \Rightarrow 0 \cdot b \le a \cdot b$

iv. If $a \le 0$, $0 \le -a \Rightarrow 0 \cdot (-a) \le (-a)(-a) \Rightarrow 0 \le a^2$

If $a \ge 0$, $a \cdot a \ge 0 \cdot a \Rightarrow a^2 \ge 0$

v. $1 = 1 \cdot 1 = 1^2 \ge 0 \Rightarrow 1^2 \cdot 1^{-1} \ge 0 \cdot 1^{-1} \Rightarrow 1 \ge 0$ $1 \ne 0$

vi. $\cancel{a \cdot a^{-1} = 1 > 0}$

$\cancel{a \cdot a^{-1} > 0,\ a > 0 \Rightarrow a^{-1}}$

Suppose $0 < a$ but $0 \not< a^{-1}$, then $a^{-1} \le 0$. $a \cdot a^{-1} \le 0 \Rightarrow 1 \le 0$ X

vii. $0 < a < b \Rightarrow 0 < a^{-1} \Rightarrow 0 < b^{-1}$

$a < b \Rightarrow a \cdot a^{-1} < b \cdot a^{-1} \Rightarrow 1 < b \cdot a^{-1} \Rightarrow 1 \cdot b^{-1} < b \cdot a^{-1} \cdot b^{-1}$

$\Rightarrow b^{-1} < a^{-1}$