

Math 104 HW 11

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1 34.2

1.1 a

$$\lim_{x \rightarrow 0} \frac{1}{x} \int_0^x e^{t^2} = \lim_{x \rightarrow 0} \frac{F(x) - F(0)}{x} = F'(0) = e^{0^2} = 1$$

1.2 b

$$\lim_{h \rightarrow 0} \frac{1}{h} \int_3^{3+h} e^{t^2} = \lim_{h \rightarrow 0} \frac{F(h+3) - F(3)}{h} = F'(3) = e^{3^2} = e^9$$

2 34.5

$$\int_{x-1}^{x+1} f(t) dt = \int_0^{x+1} f(t) dt - \int_0^{x-1} f(t) dt$$

Because f is continuous on \mathbb{R} , $\int_0^{x+1} f(t) dt$ and $\int_0^{x-1} f(t) dt$ are differentiable on \mathbb{R}

$$F'(x) = f(x+1) - f(x-1)$$

3 34.7

Let $u = x^2$

$$du = 2x dx$$

$$\int_0^1 x \sqrt{1-x^2} dx = \frac{1}{2} \int_0^1 \sqrt{1-u} du = -\frac{1}{3} (1-u)^{\frac{3}{2}} \Big|_0^1 = -\frac{1}{3} (0-1) = \frac{1}{3}$$