Math 104 Homework 11

Jonathan Guo

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Contents

1 Q 1: Ross 34.2

1.1 Part A

Let F be the antiderivative of e^{t^2}. Then, notice that

$$\lim_{x\to 0} \frac{1}{x} \int_0^x e^{t^2}\, dt = \lim_{x\to 0} \frac{F(x) - F(0)}{x - 0}$$

This is just the derivative of F at 0. The derivative of F is just e^{t^2} and if you plug in 0 you get 1.

1.2 Part B

Let F be the antiderivative of e^{t^2}. Then, notice that

$$\lim_{h\to 0} \frac{1}{h} \int_3^{3+h} e^{t^2}\, dt = \lim_{h\to 0} \frac{F(3+h) - F(h)}{3 + h - h}$$

This is just the derivative of F at 3. The derivative of F is just e^{t^2} and if you plug in 3 you get e^9.

2 Q 2: Ross 34.5

We want to show that

$$\lim_{h\to 0} \frac{F(x+h)-F(x)}{h}$$

exists. Let G be the antiderivative of f. Then, we have $F(x+h) = G(x+h+1) - G(x+h-1)$ and $F(x) = G(x+1) - G(x-1)$. So the above limit becomes

$$\lim_{h\to 0} \frac{G(x+h+1) - G(x+h-1) - G(x+1) + G(x-1)}{h}$$

Rearranging terms, we get the limit is

$$\lim_{h\to 0} \frac{G(x+h+1) - G(x+1)}{h} - \lim_{h\to 0} \frac{G(x+h-1) - G(x-1)}{h}$$

The term on the left is just the derivative of G at $x+1$, which exists since we know it is $f(x+1)$. The term on the right is just the derivative of G at $x-1$, which exists since we know it is $f(x-1)$. Therefore the original limit exists and therefore F is differentiable and the derivative is equal to

$$f(x+1) - f(x-1)$$

3 Q 3: Ross 34.7

Let $u = 1 - x^2$. So $du = -2x\,dx$. The bounds are now from 1 to 0. So we have

$$\int_0^1 x\sqrt{1-x^2}\,dx = \int_1^0 -0.5\sqrt{u}\,du = 0.5\int_0^1 \sqrt{u}\,du = 0.5\left[\frac{u^{3/2}}{1.5}\right]_0^1$$

$$= \boxed{\frac{1}{3}}$$