. If X and Y are open cover compact, prove $X \times Y$ is open cover compact

Let X and Y be open cover compact sets. Then there exists finite open covers such that
~~$X \subset \{U_n\}$ and~~
$X \subset U_1 \cup U_2 \cup \ldots U_n$ and
$Y \subset N_1 \cup N_2 \cup \ldots N_m$
Let $U_1 = (a_1, x)$, $U_n = (x, b_1)$
note x is an irrelevant value above and below
Further let $N_1 = (a_2, x)$, $N_m = (x, b_2)$
we observe the cartesian product of the finite open covers of X and Y will contain what will also be finite subcovers

$X \times Y \subset K_1 \cup \ldots \cup K_{n \times m}$

such that $K_1 = (\min(a_1, a_2), x)$
and $K_{n \times m} = (x, \max(b_1, b_2))$
~~such that the~~
thus by construction $X \times Y$ is open cover compact.

2) Let $f: X \to Y$ be a continuous map between metric spaces. Let $A \subset X$ be a subset. Decide if the following are true.

a) if A is open, then $f(A)$ is open

False, let f be any constant function then $f(A)$ will consist of only one element and will be closed.

b) If A is closed, then $f(A)$ is closed
False

c) If A is bounded then $f(A)$ is bounded
False, consider $f(x) = \frac{1}{x}$ over the reals and let $A = (0, 1]$, then $f(A) = [1, \infty)$ which is unbounded.

d). If A is compact, then f(A) is compact

True by Thm 4.14 of Rudin
continuous mappings of compact metric spaces are compact

e) If A is connected, then f(A) is connected

True by Thm 4.22 of Rudin

3) Prove there is not a continuous map $f:[0,1] \to \mathbb{R}$ such that f is surjective

If such a mapping was surjective then $f([0,1])$ would be the set $\mathbb{R}$ itself, from 4.14 of Bartle this would imply that $\mathbb{R}$ is a compact set, which is a contradiction, thus no such mapping can exist.