

$$x = \sqrt{4+2\sqrt{3}} - \sqrt{3}$$

$$x + \sqrt{3} = \sqrt{4+2\sqrt{3}}$$

$$(x + \sqrt{3})^2 = 4 + 2\sqrt{3}$$

$$x^2 + 2\sqrt{3}x + 3 = 4 + 2\sqrt{3}$$

$$x^2 + 2\sqrt{3}x - 1 - 2\sqrt{3} = 0$$

$$x^2 + 2\sqrt{3}x - (1 + 2\sqrt{3}) = 0$$

Rational solutions: $\frac{p}{q} = \frac{\pm(1+2\sqrt{3})}{\pm 1}$
 $= \pm(1+2\sqrt{3})$

$$(x + \sqrt{3})(x + \sqrt{3})$$

$$x^2 + \sqrt{3}x + \sqrt{3}x + 3$$

$$x^2 + 2\sqrt{3}x + 3$$

$$2\sqrt{3} \cdot (-\sqrt{3})$$

$$4 + 2\sqrt{3} - 2\sqrt{3}\sqrt{4+2\sqrt{3}} + 3 + 2\sqrt{3}(\sqrt{4+2\sqrt{3}} - \sqrt{3}) + 3 = 4 + 2\sqrt{3}$$

$$4 + 2\sqrt{3} - \cancel{2\sqrt{3}\sqrt{4+2\sqrt{3}}} + 3 + \cancel{2\sqrt{3}\sqrt{4+2\sqrt{3}}} - 6 + 3 = 4 + 2\sqrt{3}$$

$$4 + 2\sqrt{3} + 0 = 4 + 2\sqrt{3}$$

$$4 + 2\sqrt{3} = 4 + 2\sqrt{3} \quad \checkmark$$

x is a rational solution

x is rational

$$2.7b) \sqrt{6+4\sqrt{2}} - \sqrt{2}$$

$$x = \sqrt{6+4\sqrt{2}} - \sqrt{2}$$

$$x + \sqrt{2} = \sqrt{6+4\sqrt{2}}$$

$$(x + \sqrt{2})^2 = 6 + 4\sqrt{2}$$

$$x^2 + 2\sqrt{2}x + 2 = 6 + 4\sqrt{2} \quad \text{plug } x = \sqrt{6+4\sqrt{2}} - \sqrt{2}$$

$$6 + 4\sqrt{2} - 2\sqrt{2}\sqrt{6+4\sqrt{2}} + 2 + 2\sqrt{2}(\sqrt{6+4\sqrt{2}} - \sqrt{2}) + 2 \stackrel{?}{=} 6 + 4\sqrt{2}$$

$$6 + 4\sqrt{2} - \cancel{2\sqrt{2}\sqrt{6+4\sqrt{2}}} + 2 + \cancel{2\sqrt{2}\sqrt{6+4\sqrt{2}}} - 4 + 2 \stackrel{?}{=} 6 + 4\sqrt{2}$$

$$6 + 4\sqrt{2} + \cancel{2} - \cancel{4} \stackrel{?}{=} 6 + 4\sqrt{2}$$

$$6 + 4\sqrt{2} = 6 + 4\sqrt{2} \quad \checkmark$$

is a rational solution

thus $\sqrt{6+4\sqrt{2}} - \sqrt{2}$ is rational

3.6 a) Prove $|a+b+c| \leq |a|+|b|+|c| \quad \forall a, b, c \in \mathbb{R}$
by Δ inequality $|a+b| \leq |a|+|b| \quad |b+c| \leq |b|+|c|$

$$\begin{aligned} & |a+b+c| \\ &= |(a+b)+c| \leq |a+b|+|c| \leq |a|+|b|+|c| \end{aligned}$$

thus $|a+b+c| \leq |a|+|b|+|c|$



4.11 Consider $a, b \in \mathbb{R}$ where $a < b$. Use Density of \mathbb{Q} 4.7 to show there are infinitely many rationals between a and b .

Let a, b be two real numbers with $a < b$

then by the Density of \mathbb{Q} some rational

r exists such that $a < r < b$ where $r = \frac{m}{n}$ $m, n \in \mathbb{Z}$

Now, consider the fact we have two pairs of real numbers

where $a < r$ and $r < b$, using the Completeness

of \mathbb{Q} yet again shows that two more rationals r' and r''

exist such that $a < r' < r < r'' < b$, following this

Pattern,

infinitely many rationals exist

between a and b . \square

4.14. Let A and B be nonempty bounded subsets of \mathbb{R}
and let $A+B$ be the set of all sums $a+b$ where $a \in A, b \in B$

$$\geq) \sup(A) \geq a \quad \forall a \in A$$

$$\sup B \geq b \quad \forall b \in B$$

$$\sup(A) + \sup B \geq a+b \quad \forall a \in A, \forall b \in B$$

$a+b \quad \forall a \in A, \forall b \in B$ is defined
to be the set $A+B$ as desired
we also know $\sup(A+B)$ exists
because $(A+B)$ is a bounded
subset of \mathbb{R} , thus

$$\sup A + \sup B \geq \sup(A+B)$$

Both sides of the inequality
have been shown, thus

$$\sup A + \sup B = \sup(A+B)$$

a) \leq) (claim: $\forall b \in B, \sup(A+B) - b$ is an upper bound of A)

$$\text{WTS: } \sup(A+B) - b \geq a$$

$$\sup(A+B) \geq a+b \quad \forall a \in A, \forall b \in B$$

$A+B$ is defined as the set of all sums
 $a+b$ and necessarily the inequality is true.

$$\text{Thus } \sup(A+B) - b \geq a \quad \forall a \in A$$

$$\rightarrow \sup(A+B) - b \geq \sup A$$

$$\rightarrow \sup(A+B) - \sup A \geq b \quad \forall b \in B$$

$$\rightarrow \sup(A+B) - \sup A \geq \sup B$$

$$\rightarrow -\sup A - \sup B \geq -\sup(A+B)$$

$$\rightarrow \sup A + \sup B \leq \sup(A+B)$$

4.14 b) Prove $\inf(A+B) = \inf A + \inf B$

\leq) $\inf(A+B) \leq a+b \quad \forall a \in A, \forall b \in B$ by def

$$\inf(A+B) - b \leq a \quad \forall a \in A \rightarrow \inf(A+B) - b \leq \inf A$$

Similarly, $\inf(A+B) - a \leq b \rightarrow \inf(A+B) - a \leq \inf B$

$$\rightarrow \inf(A+B) \leq \inf A + \inf B$$

\geq) $\inf A \leq a \quad \forall a \in A, \inf B \leq b \quad \forall b \in B$

$A+B$ is the set made of all
sums of $a+b$ from A and B .

So the greatest lower bound for $A+B$

must be greater or equal the
bounds of our two subsets

$$\inf(A+B) \geq \inf A + \inf B$$

thus

$$\inf(A+B) = \inf A + \inf B$$

7.6 Determine Limits

$$a) \lim S_n \quad S_n = \frac{\sqrt{n^2+1} - n}{(\sqrt{n^2+1} + n)}$$

$$= \frac{n^2+1 - n^2}{\sqrt{n^2+1} + n}$$

$$= \lim \frac{1}{\sqrt{n^2+1} + n}$$

$$= \frac{\lim 1}{\lim \sqrt{n^2+1} + \lim n} = \frac{1}{\infty} = 0$$

$$c) \lim (\sqrt{4n^2+n} - 2n) \left(\frac{\sqrt{4n^2+n} + 2n}{\sqrt{4n^2+n} + 2n} \right)$$

$$\lim \frac{4n^2+n - 4n^2}{\sqrt{4n^2+n} + 2n} = \lim \frac{n}{\sqrt{4n^2+n} + 2n}$$

$$= \lim \frac{1}{\sqrt{4+\frac{1}{n}} + 2}$$

$$= \lim \frac{1}{4} = \frac{1}{4}$$

$$\lim \frac{\sqrt{4n^2+n}}{n} + 2 = \lim \sqrt{\frac{4n^2+n}{n^2}} + 2$$

$$= \lim \sqrt{4+\frac{1}{n}} + 2 \quad \lim \frac{1}{n} = 0$$

$$= \lim \sqrt{4+0} + 2$$

$$= 2+2=4$$

$$b) \lim (\sqrt{n^2+n} - n)$$

$$= \lim (\sqrt{n^2+n} - n) \left(\frac{\sqrt{n^2+n} + n}{\sqrt{n^2+n} + n} \right)$$

$$= \lim \frac{n^2+n - n^2}{\sqrt{n^2+n} + n}$$

$$= \lim \frac{1}{\frac{1}{n}\sqrt{n^2+n} + 1}$$

$$= \frac{\lim 1}{\lim \frac{\sqrt{n^2+n}}{n} + 1}$$

$$= \frac{\lim 1}{\lim 2} = \frac{1}{2}$$

$$\rightarrow \lim \frac{\sqrt{n^2+n}}{n} + 1$$

$$= \lim \sqrt{\frac{n^2+n}{n^2}} + 1$$

$$= \lim \sqrt{1+\frac{1}{n}} + 1$$

$$\lim \frac{1}{n} = 0$$

$$= \lim (\sqrt{1} + 1) = 2$$

1.10

$$\sum_{i=1}^n 2i + (2i-1) = 3n^2$$

Proof via induction

Base Case: $n=1$

$$2(1) + 2(1)-1 = 3(1)^2$$

$$2+2-1 = 3=3 \quad \checkmark$$

Inductive step:

$$\begin{aligned} & \sum_{i=1}^{n+1} 2(n+1) + 2i-1 \\ &= \sum_{i=1}^n (2(n+1) + 2i-1) + 4n+2 + 2(n+1)-1 \\ &= \sum_{i=1}^n (2(n+1) + 2i-1) + 4n+2 + 2n+2-1 \\ &= \sum_{i=1}^n (2(n+1) + 2i-1) + 6n+3 \stackrel{?}{=} 3(n+1)^2 \\ &= 3n^2 + 6n+3 = 3[n^2 + 2n+1] \\ &= 3n^2 + 6n+3 \end{aligned}$$



By I.H

1.12

For $n \in \mathbb{N}$, let $n!$ denote product

of n consecutive terms: $1 \cdot 2 \cdot 3 \cdot \dots \cdot n$

Let $0! = 1$ and define

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \quad \text{for } k=0, 1, \dots, n.$$

(1.1) Binomial Theorem asserts that

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \dots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

$$= a^n + na^{n-1}b + \frac{1}{2}n(n-1)a^{n-2}b^2 + \dots + n a b^{n-1} + b^n$$

Verify by induction for $n=1, 2, 3, \dots$

$$(a+b)^1 = a+b = \binom{1}{0}a + \binom{1}{1}ab$$

$$= a+b \quad \checkmark$$

$n=2$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\begin{aligned} &= \binom{2}{0}a^2 + \binom{2}{1}ab + \binom{2}{2}a^0b^2 \\ &= a^2 + 2ab + b^2 \quad \checkmark \end{aligned}$$

Similarly we have $k!(n-k)!(n-k)!$

From

1.12

a) $n=3$

$$\begin{aligned} (a^3 + 2ab + b^2)(a+b) &= a^3 + 2a^2b + ab^2 \\ &\quad + a^2b + 2ab^2 + b^3 \\ &= a^3 + 3a^2b + 3ab^2 + b^3 \\ &\stackrel{?}{=} \binom{3}{0}a^3 + \binom{3}{1}a^2b + \binom{3}{2}ab^2 + \binom{3}{3}b^3 \\ &= a^3 + 3a^2b + 3ab^2 + b^3 \quad \checkmark \end{aligned}$$

b) Show $\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$ for $k=1, 2, \dots, n$

Base case: $k=1$

$$\binom{n}{1} + \binom{n}{0} = \binom{n+1}{1}$$

Assume $\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$

holds for all k , in

the $k-1$ case we have

$$\frac{n!}{1!(n-1)!} + \frac{n!}{0!n!} = \frac{(n+1)!}{1!n!}$$

$$\frac{n \cdot (n-1)!}{(n-1)!} + \frac{1}{1} = \frac{(n+1)n!}{(n-1)!}$$

$$n+1 = n+1 \quad \checkmark$$

Alternatively: $\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$

$$\frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n-k+1)!} = \frac{(n+1)!}{k!(n+1-k)!}$$

$$\frac{n!}{k!(n-k) \cdot (n-k-1)!} + \frac{n!}{(k-1)!(n-k-1)!} = \frac{(n+1)!}{k!(n+1-k)(n-k)!}$$

$$\frac{n!}{k!(n-k)} + \frac{n!}{(k-1)!} = \frac{(n+1)!}{k!(n-k)(n-k)}$$

Bem

5115
5 = n/2

$$\frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!} = \frac{n \cdot n!}{k! \cdot (n-k)!} = \frac{n \cdot n!}{k! \cdot (n-k)! \cdot (n-k)} = \frac{n \cdot n!}{k! \cdot (n-k)! \cdot (n-k)}$$

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100. 101. 102. 103. 104. 105. 106. 107. 108. 109. 110. 111. 112. 113. 114. 115. 116. 117. 118. 119. 120. 121. 122. 123. 124. 125. 126. 127. 128. 129. 130. 131. 132. 133. 134. 135. 136. 137. 138. 139. 140. 141. 142. 143. 144. 145. 146. 147. 148. 149. 150. 151. 152. 153. 154. 155. 156. 157. 158. 159. 160. 161. 162. 163. 164. 165. 166. 167. 168. 169. 170. 171. 172. 173. 174. 175. 176. 177. 178. 179. 180. 181. 182. 183. 184. 185. 186. 187. 188. 189. 190. 191. 192. 193. 194. 195. 196. 197. 198. 199. 200. 201. 202. 203. 204. 205. 206. 207. 208. 209. 210. 211. 212. 213. 214. 215. 216. 217. 218. 219. 220. 221. 222. 223. 224. 225. 226. 227. 228. 229. 230. 231. 232. 233. 234. 235. 236. 237. 238. 239. 240. 241. 242. 243. 244. 245. 246. 247. 248. 249. 250. 251. 252. 253. 254. 255. 256. 257. 258. 259. 260. 261. 262. 263. 264. 265. 266. 267. 268. 269. 270. 271. 272. 273. 274. 275. 276. 277. 278. 279. 280. 281. 282. 283. 284. 285. 286. 287. 288. 289. 290. 291. 292. 293. 294. 295. 296. 297. 298. 299. 300. 301. 302. 303. 304. 305. 306. 307. 308. 309. 310. 311. 312. 313. 314. 315. 316. 317. 318. 319. 320. 321. 322. 323. 324. 325. 326. 327. 328. 329. 330. 331. 332. 333. 334. 335. 336. 337. 338. 339. 340. 341. 342. 343. 344. 345. 346. 347. 348. 349. 350. 351. 352. 353. 354. 355. 356. 357. 358. 359. 360. 361. 362. 363. 364. 365. 366. 367. 368. 369. 370. 371. 372. 373. 374. 375. 376. 377. 378. 379. 380. 381. 382. 383. 384. 385. 386. 387. 388. 389. 390. 391. 392. 393. 394. 395. 396. 397. 398. 399. 400. 401. 402. 403. 404. 405. 406. 407. 408. 409. 410. 411. 412. 413. 414. 415. 416. 417. 418. 419. 420. 421. 422. 423. 424. 425. 426. 427. 428. 429. 430. 431. 432. 433. 434. 435. 436. 437. 438. 439. 440. 441. 442. 443. 444. 445. 446. 447. 448. 449. 450. 451. 452. 453. 454. 455. 456. 457. 458. 459. 460. 461. 462. 463. 464. 465. 466. 467. 468. 469. 470. 471. 472. 473. 474. 475. 476. 477. 478. 479. 480. 481. 482. 483. 484. 485. 486. 487. 488. 489. 490. 491. 492. 493. 494. 495. 496. 497. 498. 499. 500. 501. 502. 503. 504. 505. 506. 507. 508. 509. 510. 511. 512. 513. 514. 515. 516. 517. 518. 519. 520. 521. 522. 523. 524. 525. 526. 527. 528. 529. 530. 531. 532. 533. 534. 535. 536. 537. 538. 539. 540. 541. 542. 543. 544. 545. 546. 547. 548. 549. 550. 551. 552. 553. 554. 555. 556. 557. 558. 559. 560. 561. 562. 563. 564. 565. 566. 567. 568. 569. 570. 571. 572. 573. 574. 575. 576. 577. 578. 579. 580. 581. 582. 583. 584. 585. 586. 587. 588. 589. 590. 591. 592. 593. 594. 595. 596. 597. 598. 599. 600. 601. 602. 603. 604. 605. 606. 607. 608. 609. 610. 611. 612. 613. 614. 615. 616. 617. 618. 619. 620. 621. 622. 623. 624. 625. 626. 627. 628. 629. 630. 631. 632. 633. 634. 635. 636. 637. 638. 639. 640. 641. 642. 643. 644. 645. 646. 647. 648. 649. 650. 651. 652. 653. 654. 655. 656. 657. 658. 659. 660. 661. 662. 663. 664. 665. 666. 667. 668. 669. 670. 671. 672. 673. 674. 675. 676. 677. 678. 679. 680. 681. 682. 683. 684. 685. 686. 687. 688. 689. 690. 691. 692. 693. 694. 695. 696. 697. 698. 699. 700. 701. 702. 703. 704. 705. 706. 707. 708. 709. 710. 711. 712. 713. 714. 715. 716. 717. 718. 719. 720. 721. 722. 723. 724. 725. 726. 727. 728. 729. 730. 731. 732. 733. 734. 735. 736. 737. 738. 739. 740. 741. 742. 743. 744. 745. 746. 747. 748. 749. 750. 751. 752. 753. 754. 755. 756. 757. 758. 759. 760. 761. 762. 763. 764. 765. 766. 767. 768. 769. 770. 771. 772. 773. 774. 775. 776. 777. 778. 779. 780. 781. 782. 783. 784. 785. 786. 787. 788. 789. 790. 791. 792. 793. 794. 795. 796. 797. 798. 799. 800. 801. 802. 803. 804. 805. 806. 807. 808. 809. 810. 811. 812. 813. 814. 815. 816. 817. 818. 819. 820. 821. 822. 823. 824. 825. 826. 827. 828. 829. 830. 831. 832. 833. 834. 835. 836. 837. 838. 839. 840. 841. 842. 843. 844. 845. 846. 847. 848. 849. 850. 851. 852. 853. 854. 855. 856. 857. 858. 859. 860. 861. 862. 863. 864. 865. 866. 867. 868. 869. 870. 871. 872. 873. 874. 875. 876. 877. 878. 879. 880. 881. 882. 883. 884. 885. 886. 887. 888. 889. 890. 891. 892. 893. 894. 895. 896. 897. 898. 899. 900. 901. 902. 903. 904. 905. 906. 907. 908. 909. 910. 911. 912. 913. 914. 915. 916. 917. 918. 919. 920. 921. 922. 923. 924. 925. 926. 927. 928. 929. 930. 931. 932. 933. 934. 935. 936. 937. 938. 939. 940. 941. 942. 943. 944. 945. 946. 947. 948. 949. 950. 951. 952. 953. 954. 955. 956. 957. 958. 959. 960. 961. 962. 963. 964. 965. 966. 967. 968. 969. 970. 971. 972. 973. 974. 975. 976. 977. 978. 979. 980. 981. 982. 983. 984. 985. 986. 987. 988. 989. 990. 991. 992. 993. 994. 995. 996. 997. 998. 999. 1000.

$$\frac{k!}{k!(n-k)!} + \frac{1}{(k-1)!} = \frac{n}{k!(n-k)!} = \frac{n}{k!(n-k)! \cdot (n-k)} = \frac{n}{k!(n-k)! \cdot (n-k)}$$

$$\frac{k!}{k!(n-k)!} + \frac{k \cdot (k-1)!}{(k-1)!} = \frac{n}{(n-k)! \cdot (n-k)}$$

$$\frac{k!}{n-k} + k = \frac{n}{(n-k)! \cdot (n-k)}$$

$$1 + k(n-k) = \frac{n}{(n-k)!}$$

$$1 + kn - k^2 = \frac{n}{n-k}$$

$$(1 + kn - k^2)(n-k) = n$$

$$n+1-k + kn^2 + kn - k^2n - k^2n - k^2 + k^3 = n$$

$$n+1-k + kn^2 + kn - k^2n - k^2n - k^2 + k^3 = n$$

$$n+1-k + kn^2 + kn - 2k^2n - k^2 + k^3 = n$$

$$\frac{k!}{k!(n-k)!} + \frac{(k-1)!}{(k-1)!} = \frac{n}{(n-k)!}$$

$$\frac{k!}{k!(n-k)!} + \frac{(k-1)!}{(k-1)!} = \frac{n}{(n-k)!}$$

$$\frac{k!}{k!(n-k)!} + \frac{(k-1)!}{(k-1)!} = \frac{n}{(n-k)!}$$

$$\frac{k!}{k!(n-k)!} + \frac{(k-1)!}{(k-1)!} = \frac{n}{(n-k)!}$$

c) Prove bin thm using induction and b)

Prove:

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

$$= a^n + na^{n-1}b + \frac{1}{2}n(n-1)a^{n-2}b^2 + \dots + nab^{n-1} + b^n$$

$$(a+b)^1 = \binom{1}{0}a^1 + \binom{1}{1}a^0b$$

$$a+b = a+b \quad \checkmark$$

Assume P(n) holds for all n

For P(n+1) we have

$$(a+b)^{n+1} = \binom{n+1}{0}a^{n+1} + \binom{n+1}{1}a^n b + \binom{n+1}{2}a^{n-2}b^2 + \dots + \binom{n+1}{n}a b^n + \binom{n+1}{n+1}b^{n+1}$$

$$(a+b)^{n+1} = a^{n+1} + \binom{n+1}{1}a^n b + \dots + \binom{n+1}{n}a b^n + b^{n+1}$$

$$= a^{n+1} + \left(1 + \binom{n}{1}\right)a^n b + \dots + \left(\binom{n}{1} + \binom{n}{2}\right)a^{n-1}b^2 + \dots + \left(\binom{n}{n-1} + 1\right)ab^n + b^{n+1}$$

From b) we know $\binom{n}{i} + \binom{n}{i+1} = \binom{n+1}{i+1}$
 thus every coefficient is as desired,
 this n+1 case is true, \square

$$n - (k-1) \\ n - k + 1$$

$$\text{Show } \binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$$

$$\frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n-k+1)!} = \frac{(n+1)!}{k!(n+1-k)!}$$

$$\frac{n!}{k!(n-k)!} + \frac{n!}{(k-1)!(n-k+1)!} = \frac{(n+1)!}{k!(n+1-k)!}$$

$$\frac{1}{k!(n-k)!} + \frac{1}{(k-1)!(n-k+1)!} = \frac{n+1}{k!(n+1-k)!} = \frac{n+1}{k!(n+1-k)(n-k)!}$$

$$\frac{(n+1-k)(n-k)!}{k!(n-k)!} + \frac{(n+1-k)(n-k)!}{(k-1)!(n-k)!} = \frac{n+1}{k!}$$

$$\frac{n+1-k}{k!} + \frac{1}{(k-1)!} = \frac{n+1}{k!}$$

$$n+1-k + \frac{k!}{(k-1)!} = n+1$$

$$n+1-k + \frac{k!}{(k-1)!} = n+1$$

$$n+1-k + k = n+1$$

$$n+1 = n+1$$

2.1 Show that $\sqrt{3}, \sqrt{5}, \sqrt{7}$, and $\sqrt{31}$ are not rationals

$\sqrt{3}$ is not rational
 Pf: By Rational Zeros Thm,
 $x^2 - 3 = 0$, here $a=1, c_2=-3, c_1=0$,
 and $c_0=-3$

The only rational solutions—
 could be ± 1 and ± 3 , however,
 of which are solutions to the
 polynomial, thus $\sqrt{3}$ is not rational

$\sqrt{5}$ is not rational
 Similarly, only solutions of $x^2 - 5$
 are ± 1 and ± 5 , which are not
 solutions

$\sqrt{7}$ is not rational
 Similarly, only rational solutions of
 $x^2 - 7$ are ± 1 and ± 7
 $\sqrt{24}$

Similarly only possible rational solutions
 of $x^2 - 24$ are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6$,
 $\pm 8, \pm 12, \pm 24$, none of which are solutions
 hence $\sqrt{24}$ is irrational
 Similarly for $\sqrt{31}$

2.2 show $\sqrt[3]{2}$, $\sqrt[3]{5}$, $\sqrt[4]{13}$ are irrational

$\sqrt[3]{2}$

$$x^3 - 2$$

$$\frac{p}{q} = \frac{\pm 1, \pm 2}{\pm 1}$$

, only possible rational solutions are $\pm 1, \pm 2$,

none of which satisfy the polynomial,

$\sqrt[3]{5}$

$$x^3 - 5$$

$\sqrt[3]{5} \rightarrow x^3 - 5$, only possible

rational solutions are $\pm 1, \pm 5$, none

of which are solutions, $\sqrt[3]{5}$ is

irrational

$$\sqrt[4]{13} \rightarrow x^4 - 13$$

, only possible

rational solutions are $\pm 1, \pm 13$,

none of which are solutions,

$\sqrt[4]{13}$ is irrational