# Math 104 HW 2

schel337

8/27/2020

## Ross 10.6

(a) For all $n, n+k \geq N$, then

$$\begin{aligned} |s_{n+k} - s_n| &\leq \sum_{i=0}^{k+1} |s_{n+i+1} - s_{n+i}| \\ &< \sum_{i=0}^{k+1} 2^{-n-i} \leq 2^{-N+1} \end{aligned}$$

Therefore for any $\varepsilon > 0$, I can chose $N$ such that $2^{-N+1} < \varepsilon$ and therefore $m, n \geq N$ ensures $|s_m - s_n| < \varepsilon$

(b) No, because the harmonic sum $\frac{1}{n}$ grows unboundedly (albeit slowly). So if we let $s_{n+1} = \frac{1}{n+1} + s_n$ be the harmonic sum, then for any $n > 0$ we would an $m > n$ such that $s_m - s_n > M$ for any $M$.

## Ross 11.2

1. $$\begin{aligned} a_{n_k} &= a_{2k} = 1 \\ b_{n_k} &= b_k \\ c_{n_k} &= c_k \\ d_{n_k} &= d_{k+2} \end{aligned}$$

2. For $a_n$ it is $\{-1, 1\}$. For $b_n$ it is 0. For $c_n$ it is $\infty$. For $d_n$ it is 0.

3. $$\begin{aligned} \limsup a_n &= 1 \\ \liminf a_n &= -1 \\ \liminf b_n &= \limsup b_n = 1 \\ \liminf c_n &= \limsup c_n = \infty \\ \limsup d_n &= \liminf d_n = 0 \end{aligned}$$

4. $b_n, d_n$ converge to 0. $c_n$ diverges to $+\infty$. $a_n$ does not converge or diverge (it oscillates)

5. All are bounded except $c_n$

## Ross 11.3

1.

$$
\begin{aligned}
s_{n_k} &= s_{3k} = \cos(\pi) \\
t_{n_k} &= t_k \\
u_{n_k} &= u_{2k} = (-1/2)^{2k} = (1/4)^k \\
v_{n_k} &= v_{2k} = 1 + \frac{1}{n}
\end{aligned}
$$

2. For $s_n$ it is $\{\cos(0), \cos(\pi/3), \cos(2\pi/3), \cos(\pi), \cos(4\pi/3), \cos(5\pi/3)\}$. For $t_n$ it is 0. For $u_n$ it is 0. For $v_n$ it is $\{-1, 1\}$.

3.

$$
\begin{aligned}
\limsup s_n &= \cos(0) = 1 \\
\liminf s_n &= \cos(\pi) = -1 \\
\liminf t_n &= \limsup t_n = 0 \\
\liminf u_n &= \limsup u_n = 0 \\
\limsup v_n &= 1 \\
\liminf v_n &= -1
\end{aligned}
$$

4. $t_n, u_n$ converge to 0. $s_n$ and $v_b$ do not converge or diverge (they oscillate)

5. All are bounded

## Ross 11.5

(a) It is $\{x \in \mathbb{R} : 0 \leq x \leq 1\}$

(b) $\limsup q_n = 0$, $\liminf q_n = 1$

## Limsup

A limsup is the largest upper bound on the infinite tail of the sequence. The supremum is taken over sets, where as the limsup is the limit as you ignore all of the leading terms of the sequence. One thing I find counter intuitive is that the limsup is approached by a monotone decreasing set of supremums, while I tend to think of supremums as increasing towards something as I expand a set. Another thing that sometimes confuses me is that is not linear for isntance $\limsup -a_n \neq -\limsup a_n$ for sequences such as $(-1)^n + 1$.