Problem 1

34.2

Solution

a) $\lim_{x\to 0} \frac{1}{x} \int_0^x e^{t^2} dt$

Let $F(x) = \int_0^x e^{t^2} dt$ we can substitiute the limit definition.

$F(0) = 0$

$\lim_{x\to 0} \frac{F(x)}{x} = \lim_{x\to 0} \frac{F(x)-F(0)}{x-0} = F'(0)$

By the fundamental theorem of calculus, the derivative of the integral is just the following.

$F'(0) = f(0) = e^(0^2) = 1$

b) $\lim_{h\to 0} \frac{1}{h} \int_3^{3+x} e^{t^2} dt$

Same principle, but replace approaching 0 by 3,

$F(x) = \int_3^x e^{t^2} dt, F(3) = 0$

$\lim_{h\to 0} \frac{F(3+h)}{h} = \lim_{h\to 0} \frac{F(3+h)-F(3)}{h} = F'(3)$

$F'(3) = f(3) = e^{3^2} = e^9$

Problem 2

34.5

Solution $F(x) = \int_{x-1}^{x+1} f(t)dt$

Given: f is continuous.

Prove: F is differentiable.

$\lim_{x \to 0}$

Split F into two parts relative to a constant on $c \in [x-1, x+1]$

$F(x) = \int_{x-1}^{c} f(t)dt + \int_{c}^{x+1} f(t)dt$

By the fundamental theorem of calculus, F is continuous,

$F(x) = \int_{x-1}^{c} f(t)dt + \int_{c}^{x+1} f(t)dt$

since f is continuous at c, we can show that the deriviative exists and of the following form:

$F'(x) = f(x+1) - f(x-1)$

Problem 3

34.7

Solution $\int_0^1 x\sqrt{1-x^2}dx$

let $u = 1 - x^2$

$du = -2xdx$

$= \int_{u(0)}^{u(1)} \frac{-1}{2}\sqrt{u}dx$

We can easily take the antiderivative of $\sqrt{u}$

$= \frac{-1}{2}\frac{2}{3}u^{\frac{3}{2}}|_1^0$

$= 0 - \frac{-1}{2}\frac{2}{3}1 = \frac{1}{3}$