Ross 12.10) $\Rightarrow$: $a<(s_n)<b$ where $a,b \in \mathbb{R}$

then $a < \lim_{N\to\infty} \sup(\{(s_n) : n \geq N\}) < b$

so $\limsup |s_n| < \infty$

$\Leftarrow$: Assume s_n not bounded, meaning $\forall \varepsilon > 0 \ \exists n$ st $s_n > \varepsilon$

~~Since s_n is finite $\forall$ real n,~~

~~can continue to choose ε as $1+\max(\{s_n : n<N\})$~~

Then $\sup\{|s_n| : \forall n\} = \infty$ since there isn't finite sup. $\limsup |s_n| \leq \sup\{s_n\}$ since subset so

$\limsup |s_n| = \infty$ which is proof by contrapositive.

12.12a) $\liminf \sigma_n \leq \limsup \sigma_n$ is trivial.

$\to \limsup \sigma_n \leq \limsup s_n$

Since $\sup\{s_n : n>1\} \geq \sup\{\sigma_n : n>1\}$

Nope, I spent so long thinking about it to no avail

But if

12.12b) If $\lim s_n$ exists then $\lim s_n = \limsup s_n = \liminf s_n$
so by squeeze thm, $\liminf s_n = \liminf \sigma_n = \limsup \sigma_n = \limsup s_n$

So $\lim s_n = \lim \sigma_n$

c) $s_n = (-1)^n$

$$\lim \sigma_n = 0 = \begin{cases} \frac{1}{n} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

$\frac{1}{n}$ goes to 0

14.2) a) ~~less than $\Sigma \frac{1}{n}$~~

root test $\frac{n}{(n+1)^2} \cdot \frac{n^2}{n-1} = \frac{n^3}{n^3+2n^2+n-n^2-2n-1} < 1$

So abs convergent

b) no, oscillates btwn -1 & 0

c) Yes, ratio test $\frac{3(n^2)}{3(n+1)^2} < 1$

d) Yes $3^n > n^5$ and $\Sigma \frac{n^3}{n^5}$ converges

e) Yes $n! > n^4$

f) Yes, definitely, less than $\Sigma \frac{1}{n^2}$ beyond point

g) Yes, less than $\Sigma \frac{1}{n^3}$ beyond point

14.10