10.6) a) Since $\lim_{n\to\infty} 2^{-n} = 0$ and $\lim_{n\to\infty} -2^{-n} = 0$

$-2^{-n} < S_{n+1} - S_n < 2^{-n}$

So by squeeze $\lim_{n\to\infty} S_{n+1} - S_n = 0$

b) Yes because limit still goes to zero.

11.2) a) $a_n$: n even monotone increasing and monotone decreasing $a_n$: 1 for n even

b) $b_n$: Monotone decreasing $n>0$

c) $c_n$: monotone increasing $n>0$

$d_n$: monotone decreasing $n>0$

b) $a_n$: $\{-1, 1\}$ $b_n$: $\{0\}$ $c_n$: $\{\infty\}$ $d_n$: $\{\frac{6}{7}\}$

c) $a_n$ limsup 1 limsup 0 limsup $\infty$ limsup $\frac{6}{7}$

liminf $-1$ liminf 0 liminf $\infty$ liminf $\frac{6}{7}$

d) $b_n, d_n$ converge $c_n$ diverges to $+\infty$

e) $a_n, b_n, d_n$ bounded

11.3) a) $s_n$: $6 \mid n$ is constant so monotone

b) $t_n$: monotone decreasing $n>0$

$u_n$: n even is monotone decreasing

$v_n$: n even is monotone decreasing

b) $s_n$ $\{\cos(\frac{n\pi}{3})$ for n in $0,1,2,3,4,5\}$

$t_n$ $\{0\}$

$u_n$ $\{0\}$

$v_n$ $\{-1, 1\}$

c) $\limsup s_n = 1$ $\limsup t_n = 0$ $\limsup u_n = 0$ $\limsup v_n = 1$

$\liminf s_n = -1$ $\liminf t_n = 0$ $\liminf u_n = 0$ $\liminf v_n = -1$

d) $t_n$ & $u_n$ converge. None diverge

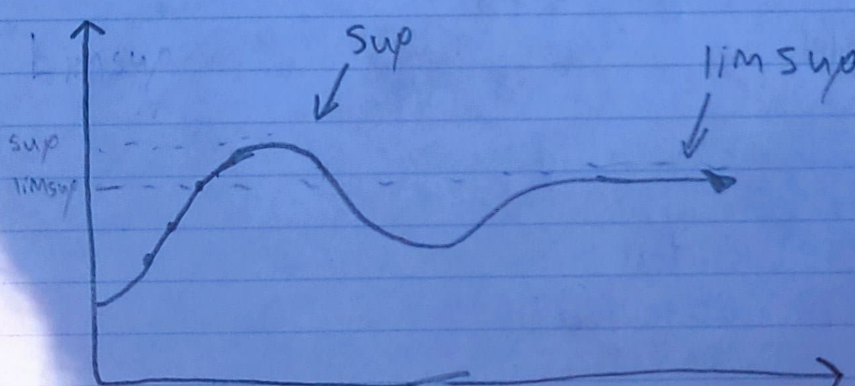
e) All are bounded

11.5) a) Any real number in $[0,1]$ because $\forall \varepsilon, \exists$ there exists infinitely many rationals in between $r$ & $r+\varepsilon$ by denseness of rationals, and by Thm 11.2 there is a subseq that converges to $r$.

b) $\limsup q_n = 1$ and $\liminf q_n = 0$ bcs else there would be rationals outside of that range which wouldn't be reached by $q_n$ bcs for a finite $N$, not all rationals in a nontrivial interval, eg) below .5 could be reached.

2) "What is up limsup?

First sup is like the maximum, except it includes values that may never be reached (effectively the lowest upper bound).




The lim sup is the limit of a sequence's sup.
In the graph above, imagine moving a curtain to cover a left portion of the graph. The limsup is the sup of the graph which is not covered, if you move the curtain to cover an arbitrarily large portion of the left part of the graph.