Fourier Series Revision Questions
Following Tao 5.1-5.5

5.1 Periodic Functions

1. Let L>0 be a real number. Define a function f that is periodic with period.
    a. Give an example of one
2. What is a Z-Periodic function?
3. In order to completely specify a Z-periodic function $f: R \to C$, in what interval must one specify f?
    a. Why? What does this determine
    b. Why can we just specify this
4. What is the name of the space of complex-valued continuous Z-periodic functions?
5. [Lemma] Basic properties of C(**R/Z;C**))
    a. Boundness
    b. Vector space and algebra properties
    c. Closure under uniform limits

5.2 Inner Product and periodic functions

1. Define the *inner product* of $f, g \in C(R/Z\ \ C))$
2. Find the inner product of $f(x) = 1\ and\ g(x) = e^{2\pi ix}$
    a. In general will the inner product be a complex or real number?
3. [Lemma] let $f, g, h \in (R/Z\ ; C)$ give the following properties
    a. Hermitian Property
    b. Positivite
    c. Linearity in the first variable
    d. Antilinearity in the second variable
4. Define the norm of f, $||f||^2$
5. Calculate the norm of the function $f(x) = e^{2\pi ix}$
6. [Lemma] $f, g \in (R/Z\ ; C$:
    a. (non-degeneracy) we have $||f||_2 = 0$ iff….
    b. (Cauchy-Schwarz inequality) we have $| < f, g > | \leq$?
    c. (Triangle in-equality)
    d. (Pythagoras' Theorem) if <f,g>=0
    e. Homogeneity
7. f,g are…. Iff <f,g>=0
8. Define the $L^2\ metric\ d_{L^2}\ on\ C(R/Z; C)$
9. The sequence f_n of functions on C(R/Z;C) will converge in the $L^2\ metric\ to\ f\ if$ …. as ..

5.3 Trigonometric Polynomials

1. Polynomials are functions of x^n (sometimes called monomials), trigonometrics are combinations of functions of … sometimes called …
2. Why is $cos(2\pi nx)$ a trig poly?
3. [Lemma] (characters are an orthonormal system).
   $For\ any\ integers\ n\ and\ m,\ we\ have\ < e_n, e_m >= 1\ when\ ...\ and\ = 0$
   $when.....\ Also\ ||e_n||\ =?$
4. [Corollary] Give the formula for the coefficients of a trig poly
5. Define Fourier Transform
   a. Give the fourier inversion formula
      i. f(x)=
   b. What is the fourier series of f?
6. Give the Plancherel formula

5.4 Periodic Convolutions

Goal is to probe the Weierstrass approximation theorem for trig poly

1. [Theorem]
   $Let\ f\epsilon\ C(R/Z\ ; C),\ and\ let\ \varepsilon > 0.\ Then\ there\ exists\ a\ trigonometric\ polynomial\ P\ st\ ...$
2. If we let P(**R\Z ; C)** denote the space of all trigonometric polynomials, then the closure of P(**R\Z;C)** in the L^∞ metric is ..
   Can be proved from the Weierstrass approx theorem for polynomials (Theorem 3.8.3)
3. Define periodic convolution $f * g: R \rightarrow C$
4. [Lemma] (Basic properties of periodic convolution)
   a. Closure
   b. Commutativity
   c. Bilinearity
5. For any $f\epsilon\ C(R/Z\ ; C)\ and\ n\ \epsilon\ Z,\ we\ have\ f * e_n\ =?$
6. Define periodic approximation to the identity

5.5 The Fourier and Plancherel Theorem

1. State the Fourier Theorem
2. [Theorem]
   $$let\ f\epsilon\ C(R/Z\ ; C,\ and\ suppose\ that\ the\ series\ \sum_{n=-\infty}^{\infty} |f^{up}(n)|\ is\ absolutely\ convergent.\ the\ the\ series...$$
3. State the Plancherel theorem