Math 105 HW 4

(25)

a) If f is measurable, Uf and $\hat{U}f$ are measurable, and $\hat{U}f \setminus Uf$ is the graph of f. Thus, $\hat{U}f \setminus Uf$ is measurable and
$$m(\hat{U}f \setminus Uf) = m(\hat{U}f) - m(Uf) = 0$$

as desired.

b) NO. Let $S \subseteq \mathbb{R}$ be a non-measurable set, and let I_S be the indicator function for S. Then, the graph of I_S is a zero set, as it is a subset of the line $y=1$ which has measure 0. However, clearly UI_S is not measurable (If it were, we could find G_δ & F_σ sets G, F s.e. $F \subset UI_S \subset G$, $m(G \setminus F) = 0$, but the projection onto $y=1$ shows that S is measurable, a contradiction.)

d) Every vertical slice of the stack exchange construction is a zero set, since this is a point. However, the function was constructed so that the graph has positive outer measure. Thus, the result of the zero-slice theorem relies on measurability; since this is the only condition violated.